QUANTUM GROUPS AND TENSOR C^*–CATEGORIES

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Quantum groups originate in the theory of Hopf algebras, which in turn has different roots: algebraic topology (Hopf, Borel, 40s); the theory of algebraic group (Dieudonné, Cartier 50s); and duality theory for locally compact groups (G.I. Kac and Takesaki 60s).

A major breakthrough was achieved by Drinfeld and Woronowicz in the mid 80s, who independently discovered important new examples as deformation of the classical Lie groups, motivated, respectively, by integrable quantum systems and low dimensional quantum field theory.

While Drinfeld used methods of Lie theory, Woronowicz emphasised an operator algebraic approach, based on the use of C^*–algebras, and motivated by Connes noncommutative geometry. We can thus study quantum groups as algebraists, geometers or analysts.

The course will try to explain these different approaches to quantum groups, and their representations. Depending on the interests, the course may, or may not, emphasise the operator algebraic approach.

In the course I shall introduce Drinfeld-Jimbo quantum groups and their representation theory. I shall consider both the case where the deformation parameter is generic and that where it is a root of unity. I shall describe Woronowicz' axiomatization of compact quantum groups, and the existence theorem of the Haar measure, as well as Peter-Weyl theory. I will introduce duality theory, and in particular Tannaka-Krein duality for quantum groups. We shall also study Woronowicz deformations of the algebra of continuous functions on the special unitary groups, $\mathrm{SU}_q(d)$, as well as the newer examples, the groups $A_u(F)$ and $A_o(F)$, introduced by S. Wang and Van Daele, which are not deformations of classical groups, and are highly noncommutative in a suitable sense. They are often referred to as the *free unitary* and *orthogonal* quantum groups, respectively. We shall discuss their representation theory, due to Banica. I shall introduce abstract tensor categories, with or without C^*–structure. Braided and ribbon categories, the important examples associated to quantum groups and relation to invariants of 3-manifolds.

References for the course are:

[1] V. Chari, A. Pressley: A guide to quantum groups (1994)
[2] C. Kassel: Quantum groups (1995).
[3] G. Lusztig: Introduction to quantum groups (1994).
[4] M. Müger: Tensor Categories: a selective guided (2010).
[5] S. Neshveyev, L. Tuset: Compact quantum groups and their representation categories (2014)
[6] T. Timmermann: An invitation to quantum groups and duality (2008).
[7] V. Turaev: Quantum invariants of knots and 3-manifolds (2010).