Introduction to quiver representations and cluster algebras

Graduate Mini-course (15 hours)

Giovanni Cerulli Irelli

When: Every Monday and Thursday at 2:30pm. First lecture April 14.

Where: Aula B, Dipartimento di Matematica. "La Sapienza"

The theory of quiver representations has been an extremely active field of research in the last 40 years. Quivers are oriented graphs that encodes the representation theory of finite dimensional algebras over an algebraically closed field. In this mini course I will establish the basics of quiver representations and prove the first results (Gabriel's theorem, Reflection functors, Auslander-Reiten theory..).

I will hence apply the acquired knowledge to categorify the theory of cluster algebras by Fomin and Zelevinsky. Recall that cluster algebras were introduced by Sergey Fomin and Andrei Zelevinsky in 2000, in order to axiomitize the behavior of many coordinate rings appearing in classical invariant theory and to provide an algebraic framework for the study of Lusztig's dual canonical basis.

Cluster algebras are commutative rings, defined by generators and relations. The generators are not given at once, but they are inductively created by a process called "mutation", starting from an initial set of variables. This notion of mutation is purely combinatorial and it is very simple and down-to-earth. Surprisingly, after the notion was introduced, it turns out to appear in many other areas of mathematics, a priori very far from each other: Poisson geometry, discrete dynamical systems, higher Teichmüller spaces, combinatorics, study of Bridgeland's stability conditions (Calabi-Yau algebras and Donaldson-Thomas invariants), Representation theory of quivers and finite dimensional algebras. Cluster algebras theory encodes the combinatorics behind these different topics, and it is now central in mathematics. By googleing "cluster algebras portal", one finds the webpage maintained by Sergey Fomin, where all the papers and new discoveries about cluster algebras are collected.

Prerequisite: I will introduce all the needed notions and no particular prerequisites are assumed.

Goal: Standard techniques to study representation theory of finite dimensional algebras will be achieved. Moreover an introduction to the combinatorics of cluster algebras will be provided. Many examples and exercises will help in understanding the general theory.

Program: The mini course consists of 15 hours.

Calendar: Tuesday and Thursday 2:30-4 pm, Aula B. Student's talks (30minutes each) will take place at the end of the course.