Dottorato in Matematica, A.A. 2013-14
SAPIENZA - Università di Roma

"CONTROLLO OTTIMO"

1. Obiettivi e organizzazione del corso

Il corso intende fornire una introduzione ai controllo ottimo e ad alcuni temi di ricerca vicini nei quali l'uso di tecniche controllistiche ha portato a risultati interessanti. Tra questi, nel programma di quest'anno figurano i metodi analitici e numerici per il controllo ottimo, la soluzione di problemi di raggiungibilità, le applicazioni alla robotica ed in ambito economico-finanziario. Il corso sarà articolato in una serie di moduli da 8 ore. Ogni parte sarà svolta da un docente diverso, ma il collegamento tra le varie parti è forte sia per quanto riguarda i concetti fondamentali che le tecniche utilizzate. Il corso si rivolge in particolare agli studenti del dottorato e dell'ultimo anno della magistrale e si inquadra nelle attività di formazione del ITN Marie Curie "SADCO - Sensitivity Analysis for Deterministic Controller Design" `http://itn-sadco.inria.fr/`. L'orario delle lezioni verrà annunciato a fine Febbraio.

I moduli previsti per quest'anno sono:
Modulo 1: Roberto Ferretti (Roma Tre)
"Monotone and Semi-Lagrangian schemes for Dynamic Programming equations"
Modulo 2: Hasnaa Zidani (ENSTA & INRIA Saclay)
"Reachability analysis for non linear systems and motion planning: Hamilton-Jacobi approach"
Modulo 3: Olivier Bokanowski (Paris 7)
"High order schemes for Hamilton-Jacobi equations"
Modulo 4: Vladimir Veliov (TU Wien)
"Optimal control applications in economics"

Data d'inizio delle lezioni: 17 Marzo 2014, ore 11
Luogo: Aula E, Dipartimento di Matematica, SAPIENZA

2. Modalità d'esame

Agli studenti sarà richiesto lo svolgimento di due tesine relative a due moduli diversi. Gli argomenti delle tesine andranno concordati con i docenti e saranno oggetto di un seminario alla fine del corso.

3. Programmi dei moduli

Monotone and Semi-Lagrangian schemes for Dynamic Programming equations

Roberto Ferretti

Dipartimento di Matematica e Fisica
Università Roma Tre

Abstract

The module is intended to present a review of the basic concepts on the approximation of PDEs of Hamilton–Jacobi–Bellman type, as well as the theory of high-order Semi-Lagrangian (SL) schemes, with a focus on finite horizon, infinite horizon and optimal stopping problems. If possible, theoretical study will be complemented by laboratory activity on simple test problems.

A rough outline of the module is the following:

- Basic facts about Dynamic Programming equations and related approximation schemes. Generalized consistency and monotonicity properties. The Barles–Souganidis convergence theorem for evolutive and stationary problems.
- Monotone schemes for evolutive problems: upwind, Lax–Friedrichs, first-order SL. Consistency, monotonicity, convergence, numerical viscosity. Boundary conditions and multi-dimensional implementation.
- Stationary and obstacle problems. Iterative solution of stationary problems: value and policy iteration.
- High-order SL schemes: construction and consistency. Lipschitz stability and quasi-monotonicity.
- Synthesis of the optimal control.

References

M. Falcone and R. Ferretti, *Semi-Lagrangian approximation schemes for linear and Hamilton–Jacobi equations*, SIAM, 2013.

Reachability analysis for non linear systems and motion Planning. Hamilton-Jacobi approach

Hasnaa Zidani
ENSTA ParisTech & Inria Saclay

Abstract

The module aims at presenting a general framework for reachability analysis and motion planning for controlled nonlinear systems in presence of obstacles. We will show that the Hamilton-Jacobi approach provides a convenient framework for both theoretical and numerical aspects. Moreover, this approach can be extended to various control problems (differential games, stochastic settings, hybrid systems, ... etc).

The outline of the module is as follows:

- Introduction to HJB approach and viscosity notions. Verification theorem. Minimum time problems. Optimal control problems with supremum running cost.

- Reachability analysis. Invariance principle. feedback control laws and motion planning. Level-set approach.

- Differential game setting. Controlled systems under uncertainties. Collision avoidance.
  - Stochastic setting with obstacles. Stochastic target problems.

References

1. A. Altarovici, O. Bokanowski, and H. Zidani, A general Hamilton-Jacobi framework for non-linear state constrained control problems, ESAIM Control Optim. Calc. Var. (2013), vol. 19(2), pp. 337–357.

2. O. Bokanowski, N. Forcadel and H. Zidani, Reachability and minimal times for state constrained nonlinear problems without any controllability assumption, SIAM J. Control and Optimization (2012), vol. 48(7), pp. 4292-4316.

High order schemes for Hamilton-Jacobi equations

Olivier Bokanowski
University Paris Diderot (Paris 7)

Abstract

This module will present the following topics:

1. High order finite difference schemes for Hamilton-Jacobi (HJ) equations. ENO schemes, RK-TVD schemes. Projection scheme. Error estimates, Krylov's technique.

2. High-order discontinuous Galerkin (DG) approaches. A direct DG method. Coupling DG and semi-Lagrangian approaches. Splitting methods. Case of an obstacle HJ equation. Case of second order HJ equations.

3. High-dimensional problems, boundary conditions, computational domain reduction technique and other cputime reduction techniques. A word about parallel programming. Examples in optimal control and for option pricing.

4. Finite difference schemes for second order HJ equation coming from stochastic optimal control problems and optimal stopping time problems. The American option toy problem. Non linear finite difference implicit schemes.

References

[1] Osher, S. and Shu, C.-W. *High order essentially non-oscillatory schemes for Hamilton-Jacobi equations*, SIAM J. Numer. Anal., Vol 28, pp. 907–922, 1991.

[2] Shu, C.-W. *High order numerical methods for time dependent Hamilton-Jacobi equations.* Mathematics and computation in imaging science and information processing, 47–91, Lect. Notes Ser. Inst. Math. Sci. Natl. Univ. Singap., 11, World Sci. Publ., Hackensack, NJ, 2007.

[3] Bokanowski, O., Cheng, Y. and Shu, C.-W., *A discontinuous Galerkin scheme for front propagation with obstacles*, Numerische Math., to appear.

[4] Bokanowski, O. and Simarmata, G., *Semi-Lagrangian discontinuous Galerkin schemes for first and second order Partial Differential Equations*, Preprint.

Optimal control applications in economics

Vladimir Veliov
TU Wien

Abstract

This module will present the following topics:

1. Various types of optimal control problems. Necessary optimality conditions: Pontryagin's maximum principle. Sufficient optimality conditions: Arrow, etc. Simple economic examples. Problems with state constraints.

2. Infinite-horizon problems. Connection with the dynamic programming approach. Some relevant facts from the dynamical systems theory (hyperbolic equilibria, saddle point stability, limit cycles).

3. Analysis of some economic models: Ramsey, extraction of renewable resources, production planning and advertising, ... Thresholds (indifference points) .

4. Models involving heterogeneity; age-structured problems. Vintage capital models, life-time planning, models involving contagious behavior.

References

1. G. Feichtinger, R.F. Hartl, P. M. Kort, V. M. Veliov, *Anticipation effects of technological progress on capital accumulation: a vintage capital approach*, Journal of Economic Theory, **126** (2006), 143164.

2. G. Feichtinger, R.F. Hartl, P. M. Kort, V. M. Veliov, *Capital accumulation under technological progress and learning: A vintage capital approach*, European Journal of Operational Research, **172** (2006), 293310.

3. G. Feichtinger, A. Prskawetz, V. M. Veliov, *Age-structured optimal control in population economics*, Theoretical Population Biology, **65** (2004), 373387.