Dottorato in Matematica, A.A. 2012-13
SAPIENZA - Università di Roma

"CONTROLLO OTTIMO"

1. Obiettivi e organizzazione del corso

Il corso intende fornire una introduzione ai controllo ottimo e ad alcuni temi di ricerca vicini nei quali l'uso di tecniche controllistiche ha portato a risultati interessanti. Tra questi, metodi analitici e numerici per il controllo predittivo, il controllo switching, la soluzione di problemi di controllo ottimo e dei giochi di campo medio. Il corso sarà articolato in una serie di moduli da 8 ore. Ogni parte sarà svolta da un docente diverso, ma il collegamento tra le varie parti è forte sia per quanto riguarda i concetti fondamentali che le tecniche utilizzate. Il corso si rivolge in particolare agli studenti del dottorato e dell'ultimo anno della magistrale e si inquadra nelle attività di formazione del ITN Marie Curie "SADCO - Sensitivity Analysis for Deterministic Controller Design" `http://itn-sadco.inria.fr/`.

I moduli previsti per quest'anno sono:
Modulo 1: Lars Grüne (Bayreuth University), "Introduction to Model Predictive Control"
Modulo 2: Umberto Mosco (Worcester Polytechnic Institute, USA), "Optimal switching for fractal growth"
Modulo 3: Emiliano Cristiani (IAC-CNR, Roma), "Metodi numerici per i problemi di controllo ottimo"
Modulo 4: Fabio Camilli (SAPIENZA), "Giochi di campo medio"

Data d'inizio delle lezioni: 3 Aprile 2013, ore 11
Luogo: Aula B, Dipartimento di Matematica, SAPIENZA

2. Modalità d'esame

Agli studenti sarà richiesto lo svolgimento di due tesine relative a due moduli diversi. Gli argomenti delle tesine andranno concordati con i docenti e saranno oggetto di un seminario alla fine del corso.

3. Programmi dei moduli

Modulo 1

Introduction to Model Predictive Control

Lars Grüne, Department of Mathematics, Bayreuth University,
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Model predictive control (MPC) is a method to synthesize discrete time infinite horizon approximately optimal feedback laws from the iterative solution of finite horizon optimal control problems. The first part of the course will cover the classical engineering problem of stabilizing optimal control problems, which is predominant in control engineering. We will present different variants of MPC schemes for this problem - particularly formulations with an without so called stabilizing terminal constraints - and investigate their stability and performance properties. In the second part of the talk we will investigate more general classes of optimal control problems which leads to the so called economic MPC problem. Again, we will present different variants with and without terminal constraints and will analyze their performance.

Essential references

1. L. Grüne, *NMPC without terminal constraints*, in Proceedings of the IFAC Conference on Nonlinear Model Predictive Control 2012 (NMPC'12), 2012, pp. 1-13, available from `http://num.math.uni-bayreuth.de/de/publications/2012/gruene_nmpc_without_terminal_constraints_2012`

2. L. Grüne and J. Pannek, Nonlinear Model Predictive Control. Theory and Algorithms. Springer, London, 2011, sample chapters (free) and full text (for subscribers) available from `http://www.nmpc-book.com`

3. D.Q. Mayne, J.B. Rawlings, C.V. Rao, P.O.M. Scokaert, *Constrained model predictive control: Stability and optimality*, Automatica, 36 (2000), pp. 789-814, available from `http://www3.imperial.ac.uk/pls/portallive/docs/1/27681700.PDF`

Modulo 2

Optimal switching for fractal growth

Umberto Mosco, Dipartimento di Matematica, Worcester University,
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A well known problem in optimal control is the optimal switching for ordinary differential equations. The problem refers to a system whose state is modeled by the solution of an ordinary differential equation in $\mathbb{R}^n$. The differential equation does not remain the same all along the evolution of the system, in fact, there is a finite number of ordinary differential operators, say operator

A and operator B, that are both candidate to govern the evolution of the system. The system can be switched from one to the other equation (that is, operator) at any time. The evolution of the system in time is then controlled by the choice of the sequence of switching times θ_i and equations $d_i \in A, B$. An optimal control is a sequence $\{\theta_i, d_i\}$ that minimizes some cost functional depending on the resulting whole trajectory

Fractal sets (and related differential operators) are produced by the iterated action of contractive similarities families. These iteration processes can be seen as a discrete time evolution of sets and operators driven by similarity.

Fractal mixtures occur when the evolution is governed jointly by a finite number of similarity families, say family A and family B. In this case the evolution is modified by switching similarities $d_i \in A, B$ all along a sequence of switching times (that is, iteration levels) θ_i.

The sequence $\{\theta_i, d_i\}$ can be considered as a control variable acting upon the fractal growth process produced jointly by the families A and B. The asymptotic fractal set and operators produced by $\{A, B\}$ become controllable by the choice of the sequence $\{\theta_i, d_i\}$.

In our lectures we provide the background for the fractal theory required in this application and we investigate the fractal control setting described before from the point of view of optimal control.

For some preliminaries about fractal mixtures we refer to [4]. . For background on optimal switching control of ordinary differential equation we refer to the book Bardi- Capuzzo Dolcetta 1. and Capuzzo Dolcetta and co-authors [2-3].

Essential references

1. M. Bardi, I. Capuzzo Dolcetta, Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations, Birkhäuser, Boston, 1997.

2. I. Capuzzo Dolcetta, L.C. Evans, *Optimal switching for ordinary differential equations*, SIAM J. on Control and Optimization, 1984

3. I. Capuzzo Dolcetta, M Matzeu, J.L Menaldi, *On a system of first-order quasi-variational inequalities connected with the optimal switching problem*, Systems & Control Letters, 3 (1983), 113116.

4. U. Mosco, *Harnack inequalities on scale irregular Sierpiǹski gaskets*, in Nonlinear Problems in Mathematical Physics and Related Topics II, In Honor of Professor O. A. Ladyzhenskaya, Edited by Birman et al., Kluwer Academic/Plenum Publishers, New York, pages 305328, 2002.

Modulo 3

Metodi numerici per i problemi di controllo ottimo (numerical methods for optimal control problems)

Emiliano Cristiani, IAC-CNR, Roma
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The course starts with a brief introduction on optimal control problems (infinite and finite horizon problem, minimum time problem), dynamic programming principle and Hamilton-Jacobi-Bellman (HJB) equations. The level will be set on the basis of students' knowledge (both first- and second-year students are welcome).

Then we investigate some classical methods for computing optimal controls and optimal trajectories. We introduce the Pontryagin Maximum Principle (PMP) as a necessary condition of optimality and we show the link to the viscosity solution of HJB equations. We present some numerical methods to approximate optimal trajectories using PMP, HJB and both. We also give some ideas about direct approximation of optimal trajectories via simpler finite-dimensional methods.

In the final part, we present some recent numerical methods to solve efficiently HJB equations. These methods overcome classical approaches based on fixed point algorithms, since they compute the solution in a few number of iterations (single-pass methods). We study the Fast Marching method (1996),the Ordered Upwind method (2003),the Fast Sweeping method (2005), the Buffered Fast Marching method (2009), and we finally give a general framework for the applicability of single-pass schemes for HJB equations.

Essential References

1. M. Bardi, I. Capuzzo Dolcetta, Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations, Birkhäuser, 1997.
2. L. C. Evans, An introduction to mathematical optimal control theory, notes.
3. J. A. Sethian, Level set methods and Fast Marching methods, Cambridge University Press, 1999.
4. A selection of recent articles on single-pass methods for HJB equations.

Modulo 4

Giochi a campo medio e modelli correlati (Mean field games and related models)

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I modelli dei giochi a campo medio (Mean Field Games), introdotti da Lasry e Lions nel 2006, sono usati per descrivere il comportamento di una popolazione composta da un numero molto grande di agenti identici che cercano di ottimizzare le loro scelte in funzione della distribuzione degli altri individui della popolazione. Da un punto di vista matematico i giochi a campo medio portano allo studio di un sistema accoppiato composto da una equazione di Hamilton-Jacobi-Bellman e da una equazione di Fokker-Planck, , che presenta una notevole complessita' da un punto di vista matematico. L'obiettivo di questo corso è da un lato di fornire un'introduzione alla problematica dei giochi a campo medio, dall'altro lato di discutere alcune applicazioni del modello considerando anche la loro risoluzione numerica.

Essential References

1. J.M. Lasry,, P.L. Lions, *Mean Field Games*, Japan J. Math. 2 (2007), no. 1, 229260.
2. J.M. Lasry,, P.L. Lions, *Jeux a champ moyen. I. Le cas stationnaire.* C. R. Math. Acad. Sci. Paris 343 (2006), no. 9, 619625.
3. J.M. Lasry,, P.L. Lions, *Jeux a champ moyen. II. Horizon fini et controle optimale*, C. R. Math. Acad. Sci. Paris 343 (2006), no. 10, 679684.
4. P. Cardaliaguet, *Notes on Mean Field Games*, Note del corso tenuto a Tor Vergata, 2010