

Course title: **SEMICLASSICAL ANALYSIS**

Duration: 16 hours

Program:¹

1. **Introduction.** Motivation: the semiclassical limit for the Schrödinger equation; eigenfunction concentration. Classical asymptotic methods: the WKB Ansatz.
2. **Background on the Fourier transform.** The stationary phase method.
3. **The Weyl quantization and the Wigner transform.** Semiclassical pseudodifferential operators. The product theorem. Positivity and adjoints. L^2 -boundedness. Wigner distributions.
4. **Wigner measures and the semiclassical limit.** Semiclassical/Wigner measures. Egorov's theorem and the Wigner equation. The semiclassical limit for Schrödinger equations with smooth coefficients.
5. **Additional topics: the semiclassical limit for non-smooth potentials.** Potentials with point singularities. Potentials with interfaces. The case of non-smooth potentials and mixed-state type initial data. Applications to the Vlasov-Poisson system.
6. **Additional topics: the semiclassical limit beyond the Ehrenfest time.** Semiclassical limits for the non-semiclassical Schrödinger group $e^{it\Delta}$. Applications to dispersive and unique continuation properties of the Schrödinger equation.
7. **Additional topics: eigenfunction concentration.** The Wigner transform on a compact manifold. The problem of classification of quantum limits. Quantum limits on the sphere. Schnirelman's theorem and the Quantum Unique Ergodicity conjecture.

References

- [1] Mouez Dimassi and Johannes Sjöstrand. *Spectral asymptotics in the semiclassical limit*, volume 268 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge, 1999.

¹One or several of the additional topics (5 to 7) can be chosen according to the preferences of the audience, and the time availability.

- [2] Lawrence C. Evans and Maciej Zworski. *Lectures on semiclassical analysis*. 2010. Available at: <http://www.math.berkeley.edu/~zworski/semiclassical.pdf>.
- [3] Gerald B. Folland. *Harmonic analysis in phase space*, volume 122 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 1989.
- [4] Pierre-Louis Lions and Thierry Paul. Sur les mesures de Wigner. *Rev. Mat. Iberoamericana*, 9(3):553–618, 1993.