GEOMETRY OF KÄHLER THREEFOLDS

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Compact Kähler manifolds are a natural generalisation of complex projective manifolds, the most well-known examples are complex tori and K3 surfaces[1]. We know that they are the natural setting for Hodge theory, moreover they are stable under small deformations. Unfortunately we are not very good at describing their geometry. In fact, the methods of Mori's minimal model program do not apply to the Kähler setting, for example we can not use a reduction to positive characteristic.

In this mini-course I will explain why the minimal model program should nevertheless exist for compact Kähler spaces and how this can actually be proven for threefolds. I will also discuss a number of applications: classification of compact Kähler threefolds without subvarieties, fundamental groups and approximation by projective varieties.

Approximate planning:

- Kähler manifolds: positivity notions and basic examples (the surface case, nef and big cohomology classes, bad contractions in threefold case)
- MMP for Kähler threefolds: statements and existence of rational curves
- Applications: classification, fundamental groups and algebraic approximation
- Idea of the proof of the contraction theorem. Challenges in higher dimension

It is not necessary to know anything about the MMP to follow the lectures, basic notions of complex algebraic geometry (line bundles, divisors, cohomology, blowups, intersection form on surfaces) are sufficient.

References

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[1] A complex torus or a K3 surface can be a projective manifold, but in some sense most of them are not.