Large deviation functions of the density and of the current for diffusive systems

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- *Fluctuations in stationary nonequilibrium states of irreversible processes.* Phys. Rev. Lett. (2001).
- *Macroscopic fluctuation theory for stationary non-equilibrium states.* J. Stat. Phys. (2002)

...

- *Current fluctuations in stochastic lattice gases.* Phys. Rev. Lett. (2005)
- *Non equilibrium current fluctuations in stochastic lattice gases.* J. Stat. Phys. (2006)

...

- Macroscopic fluctuation theory. Rev. Mod. Phys. (2015)

Bertini, De Sole, Gabrielli, Jona-Lasinio, Landim

A theory for non-equilibrium steady states

- Goal: generalize the Boltzmann entropy to non-equilibrium steady states
- Generalize second law and Clausius inequality to non-equilibrium
- Tool to calculate large deviation functions out of equilibrium (for diffusive systems)
- Works also in non steady state situations (and to calculate finite size corrections)

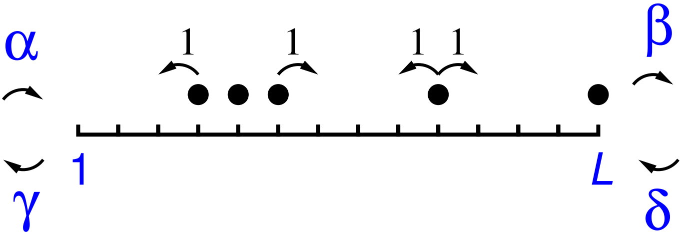
Outline of this talk

- The macroscopic fluctuation theory in a nutshell
- Current fluctuations and phase transitions in one dimension
- Current fluctuations in higher dimensions
- Recent developments

The macroscopic fluctuation theory in a nutshell

Exclusion process

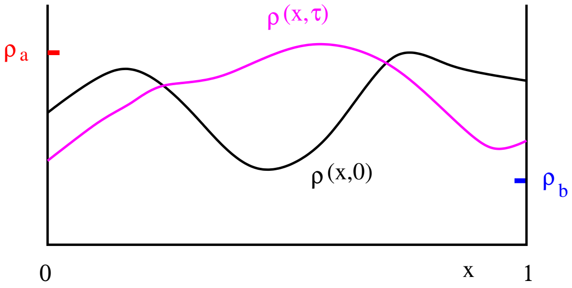
SSEP (Symmetric simple exclusion process)



$$\rho_a = \frac{\alpha}{\alpha + \gamma}, \qquad \rho_b = \frac{\delta}{\beta + \delta}$$

Macroscopic fluctuation theory

Bertini De Sole Gabrielli
Jona-Lasinio Landim 2001-2002



Diffusive scaling

$$\text{time} = L^2\ t \quad ; \quad \text{position} = L\ x; \quad \text{current} = \frac{j}{L}$$

Macroscopic fluctuation theory

Bertini De Sole Gabrielli Jona-Lasinio Landim 2001 $\rightarrow$

Evolution of a density profile $\rho(x,t)$ and a rescaled current $j(x,t)$

$$\mathrm{Pro}(\{\rho(x,t), j(x,t)\}) \sim \exp\left[-L\int_0^\tau dt \int_0^1 dx\, \frac{\left(j + D(\rho)\,\rho'\right)^2}{2\,\sigma(\rho)}\right]$$

with

- $\frac{d\rho}{dt} = -\frac{dj}{dx}$ (conservation law)
- $\rho(0,t) = \rho_a \;;\; \rho(1,t) = \rho_b$

SSEP

$$D(\rho) = 1$$

$$\sigma(\rho) = 2\rho(1-\rho)$$

Macroscopic fluctuation theory

$$\boxed{\mathrm{Pro}(\{\rho(x,t), j(x,t)\}) \sim \exp\left[-L\int_0^\tau dt\int_0^1 dx\,\frac{\left(j+D(\rho)\,\rho'\right)^2}{2\sigma(\rho)}\right]}$$

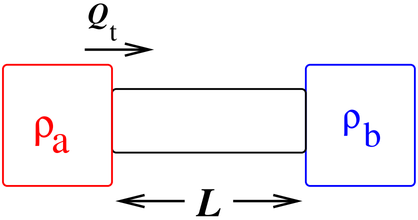
$$\Updownarrow$$

$$\boxed{j(x,t) = -D(\rho' x,t))\,\rho'(x,t) + \frac{1}{\sqrt{L}}\eta(x,t)}$$

with the white noise

$$\langle\eta(x,t)\eta(x',t')\rangle = \sigma(\rho)\,\delta(x-x')\,\delta(t-t')$$

For general diffusive systems



Diffusive system

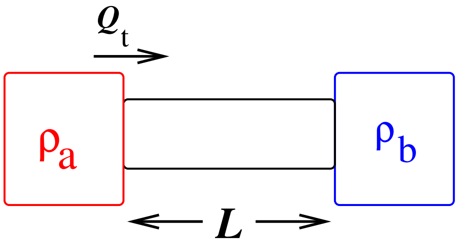
- For $\rho_a - \rho_b$ small : $\frac{\langle Q_t \rangle}{t} = \frac{D(\rho)(\rho_a - \rho_b)}{L}$

- $\rho_a = \rho_b = \rho$: $\frac{\langle Q_t^2 \rangle}{t} = \frac{\sigma(\rho)}{L}$

SSEP: $D = 1$ and $\sigma = 2\rho(1-\rho)$

Current fluctuations in $d = 1$

Current fluctuations in the steady state



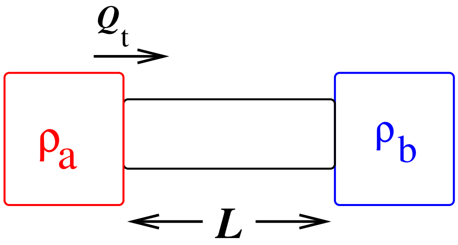
2 reservoirs of particles

$\langle Q_t \rangle$?

$\langle Q_t^2 \rangle - \langle Q_t \rangle^2$?

Distribution $P(Q_t)$ of Q_t

Current fluctuations in the steady state



2 reservoirs of particles

$\langle Q_t \rangle$?

$\langle Q_t^2 \rangle - \langle Q_t \rangle^2$?

Distribution $P(Q_t)$ of Q_t

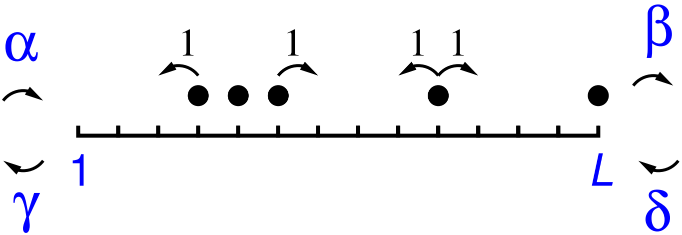
Generating function (for large t)

$$\langle \exp [\lambda Q_t] \rangle \sim \exp [t \ \mu(\lambda)]$$

($Q_t \simeq$ Sum of t/t_0 independent random variables)

Exclusion processes

SSEP (Symmetric simple exclusion process)

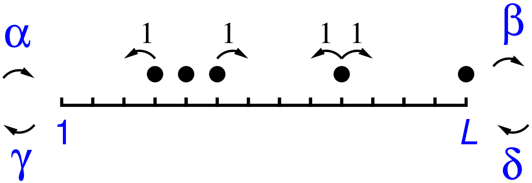


$$\rho_a = \frac{\alpha}{\alpha + \gamma} \,, \qquad \rho_b = \frac{\delta}{\beta + \delta}$$

$$\boxed{\langle \exp\left[\lambda Q_t\right] \rangle \sim \exp\left[t\ \mu(\lambda)\right]} \quad ?$$

Two approaches

SSEP (Symmetric simple exclusion process)



Microscopic

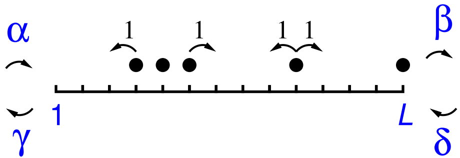
Bethe ansatz, Perturbation theory, Computer,...

Macroscopic

$$\mathrm{Pro}(\{\rho(x,t), j(x,t)\}) \sim \exp\left[-L \int_0^{\tau} dt \int_0^1 dx \, \frac{[j+\rho']^2}{4\rho(1-\rho)}\right]$$

SSEP (Symmetric simple exclusion process)

D Douçot Roche 2004

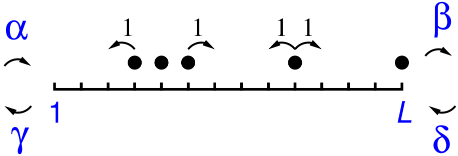


$$\lim_{t\to\infty} \frac{\langle Q(t)\rangle}{t} \simeq \frac{1}{L}[\rho_a - \rho_b] \qquad \text{Fick's law}$$

$$\lim_{t\to\infty} \frac{\langle Q^2(t)\rangle_c}{t} \simeq \frac{1}{L}\left[\rho_a + \rho_b - \frac{2(\rho_a^2 + \rho_a\rho_b + \rho_b^2)}{3}\right]$$

$$\lim_{t\to\infty} \frac{\langle Q^3(t)\rangle_c}{t} \simeq \frac{1}{L}(\rho_a - \rho_b)\left[1 - 2(\rho_a + \rho_b) + \frac{16\rho_a^2 + 28\rho_a\rho_b + 16\rho_b^2}{15}\right]$$

Current fluctuations in the SSEP



$$\boxed{\left\langle e^{\lambda Q_t} \right\rangle \sim e^{t\ \mu(\lambda)}}$$

For large L $\qquad \boxed{\mu(\lambda, \rho_a, \rho_b, \text{contacts}) = \frac{1}{L} R(\omega)}$

where

$$\omega = \rho_a\,(e^\lambda - 1) + \rho_b\,(e^{-\lambda} - 1) + \rho_a\,\rho_b\,(e^\lambda - 1)(e^{-\lambda} - 1)$$

Result: $\qquad \boxed{R(\omega) = [\log(\sqrt{1+\omega} + \sqrt{\omega})\ ]^2}$

For $\rho_a = \rho_b = \frac{1}{2}$ $\qquad R(\omega) = \dfrac{\lambda^2}{4} \quad \Leftrightarrow \quad Q_t$ is Gaussian

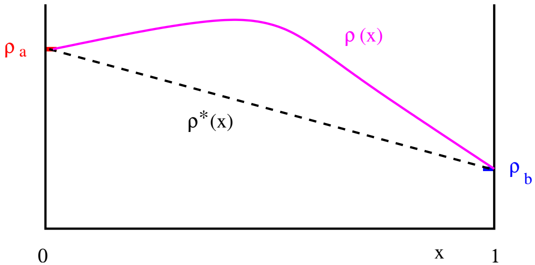
Variational principle in $d = 1$

Bodineau D 2004

Assuming that $j(x,t) = j$

Generating function $\langle e^{\lambda Q_t} \rangle \sim \exp[t\mu_L(\lambda)]$

$$\mu_L(\lambda, \rho_a, \rho_b) = \frac{1}{L} \max_{j,\rho(x)} \left[\lambda j - \int_0^1 dx \, \frac{[j\ L\ + \rho'(x)\ D(\rho(x))]^2}{2\sigma(\rho(x))} \right]$$



Cumulants for the SSEP

$$\frac{\langle Q_t^4 \rangle_c}{t} = \frac{1}{L} \; \frac{3\,(\,5\ I_4\ I_1^2 - 14\ I_1 I_2\ I_3 + \ 9\ I_2^3)}{I_1^5}$$

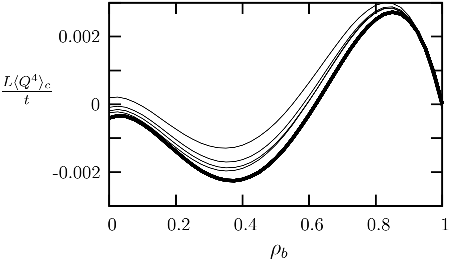
Bodineau D 2007

where

$$I_n = \int_{\rho_b}^{\rho_a} D(\rho)\sigma(\rho)^{n-1} d\rho$$

$D(\rho) = 1 \quad , \quad \sigma(\rho) = 2\rho(1-\rho)$

For $\rho_a = 1$ and $L = 5, 9, 13, 17, \infty$



True variational principle

Bertini De Sole Gabrielli
Jona-Lasinio Landim 2005

$$\mu(\lambda) = \frac{1}{L} \lim_{T\to\infty} \max_{\rho,j} \frac{1}{T} \int_0^T dt \int_0^1 dx \; \lambda j(x,t) - \frac{[j(x,t) + \rho'(x,t) D(\rho(x,t))]^2}{2\sigma(\rho(x,t))}$$

with $\frac{d\rho}{dt} = -\frac{dj}{dx}$ (conservation), $\rho_t(0) = \rho_a$, $\rho_t(1) = \rho_b$

- Sufficient condition for the optimal profile to be time independent
- Dynamical phase transition

the optimal $\rho_t(x)$ starts to become time dependent

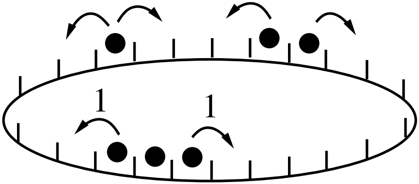
Ring geometry

Bodineau D 2005
Appert D Lecomte Van Wijland 2008

N particles

L sites

$$\rho = \frac{N}{L}$$



Q_t flux through a bond during time t

Universal cumulants of the current)

SSEP:

$$\boxed{\frac{\langle Q^2 \rangle_c}{t} = \frac{\sigma}{L}}$$

Gaussian

$$\boxed{\frac{\langle Q^4 \rangle_c}{t} \simeq \frac{\sigma^2}{2L^2}, \quad \frac{\langle Q^6 \rangle_c}{t} \simeq -\frac{\sigma^3}{4L^2}, \quad \frac{\langle Q^8 \rangle_c}{t} \simeq \frac{5\sigma^4}{12L^2}}$$

Universal

For a general diffusive system $\boxed{\left\langle e^{\lambda Q_t} \right\rangle \sim e^{t\,\mu(\lambda)}}$ with

$$\mu(\lambda) - \frac{\lambda^2 \langle Q^2 \rangle}{2t} = \frac{1}{L^2} D \mathcal{F}\left(\frac{\sigma \sigma''}{16 D^2} \lambda^2\right)$$

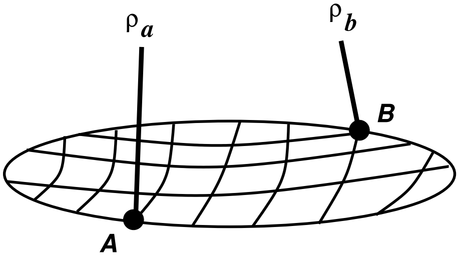
$$\mathcal{F}(u) = -4 \sum_{n \geq 1} \left[n\pi \sqrt{n^2\pi^2 - 2u} - n^2\pi^2 + u \right] = \frac{1}{3} u^2 + \frac{1}{45} u^3 + \frac{1}{378} u^4 + \ldots$$

Phase transition as $u \to \pi^2/2$ if $\sigma(\rho)$ is convex

Current fluctuations in $d > 1$

General graph

D Gerschenfeld 2009



$$\langle e^{\lambda Q_t} \rangle \sim e^{t\ \mu(\lambda)}$$

$$\mu(\lambda, \rho_a, \rho_b, \text{contacts}) = R(\omega, \text{graph}, \text{contacts})$$

where

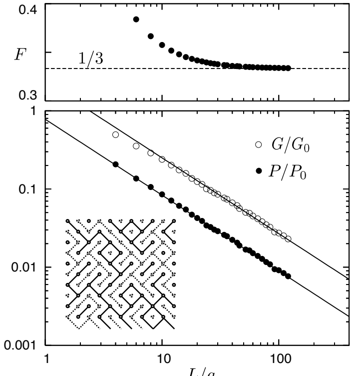
$$\omega = \rho_a\,(e^{\lambda} - 1) + \rho_b\,(e^{-\lambda} - 1) + \rho_a\,\rho_b\,(e^{\lambda} - 1)(e^{-\lambda} - 1)$$

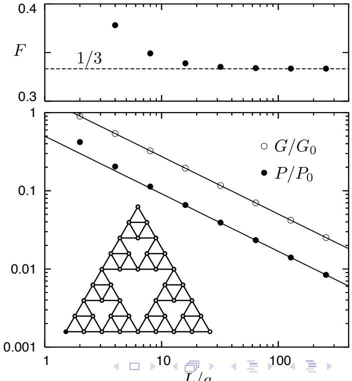
Fano Factor:

$$F = \lim_{t\to\infty} \frac{\langle Q_t^2 \rangle - \langle Q_t \rangle^2}{\langle Q_t \rangle}$$

Groth, Tworzydło, Beenakker (2008)

SSEP with $\rho_a = 1$ and $\rho_b = 0$



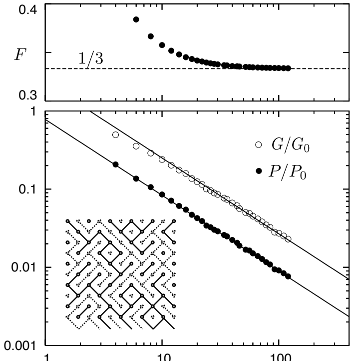


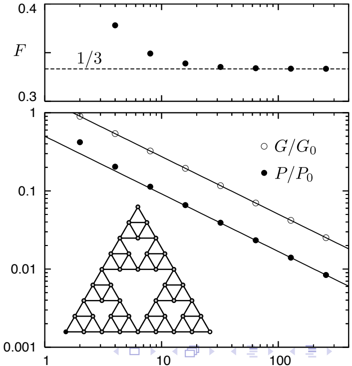
Fano factor

$$F = \lim_{t \to \infty} \frac{\langle Q_t^2 \rangle - \langle Q_t \rangle^2}{\langle Q_t \rangle} \to \frac{1}{3}$$

Groth, Tworzydło, Beenakker (2008)

SSEP with $\rho_a = 1$ and $\rho_b = 0$

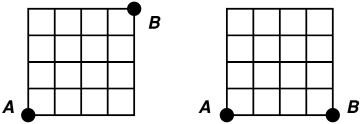


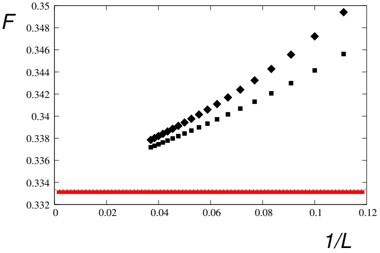


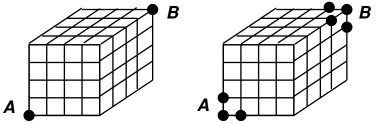
Fano factor

Akkermans Bodineau D Shpielberg (2013)

SSEP with $\rho_a = 1$ and $\rho_b = 0$





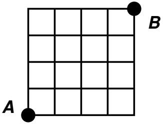


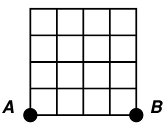
Cube 1   ***Cube 2***

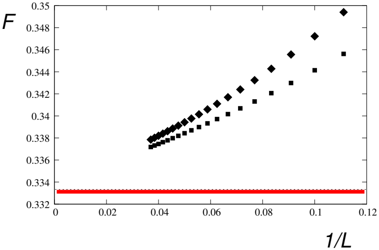
Fano factor

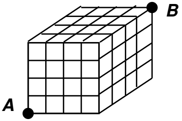
Akkermans Bodineau D Shpielberg (2013)

SSEP with $\rho_a = 1$ and $\rho_b = 0$

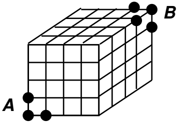




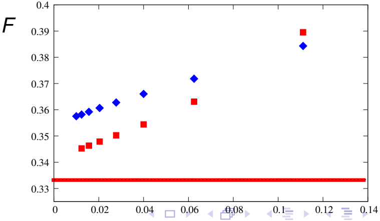




Cube 1



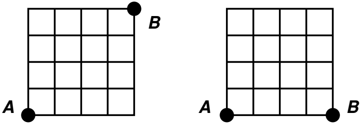
Cube 2

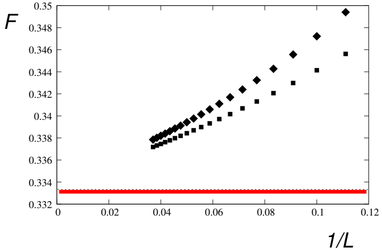


Fano factor

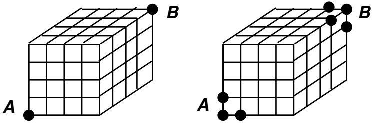
Akkermans Bodineau D Shpielberg (2013)

SSEP with $\rho_a = 1$ and $\rho_b = 0$

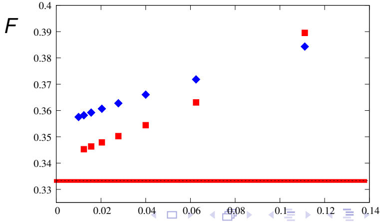




$F = \frac{1}{3}$



Cube 1 ***Cube 2***



$F \neq \frac{1}{3}$

Variational principle in higher dimension

$$\mu(\lambda) = L^{d-2} \min_{\{\vec{j}(\vec{r}),\rho(\vec{r})\}} \int d\vec{r} \; \left(-\lambda \vec{\nabla} v(\vec{r}) \, . \, \vec{j}(\vec{r}) \; - \; \frac{[\vec{j}(\vec{r}) + D(\rho(\vec{r})) \; \vec{\nabla} \rho(\vec{r}) \,]^2}{2\sigma(\rho(\vec{r}))} \right)$$

where

$$\Delta v(\vec{r}) = 0 \quad ; \quad v(\partial A) = 1 \quad \text{and} \quad v(\partial B) = 0$$

Optimization equations:

$$\vec{j}(\vec{r}) = -D(\rho(\vec{r})) \, \vec{\nabla} \rho(\vec{r}) + \sigma(\rho(\vec{r})) \vec{\nabla} H(\vec{r})$$

Equations to solve:

$$\vec{\nabla} . \Big(D(\rho(\vec{r})) \, \vec{\nabla} \rho(\vec{r}) \Big) = \vec{\nabla} . \, \Big(\sigma(\rho(\vec{r})) \, \vec{\nabla} H(\vec{r}) \Big)$$

$$D(\rho(\vec{r})) \; \Delta H(\vec{r}) = -\frac{\sigma'(\rho(\vec{r}))}{2} \left(\vec{\nabla} H(\vec{r}) \right)^2$$

with boundary conditions on ρ and H.

Solution in higher dimension

$$\Delta v(\vec{r}) = 0 \quad ; \quad v(\partial A) = 1 \quad \text{and} \quad v(\partial B) = 0$$

Solution:

$$H(\vec{r}) = H_1(v(\vec{r}))$$

$$\rho(\vec{r}) = \rho_1(v(\vec{r}))$$

and

$$\boxed{\mu(\lambda) = \kappa(L) \times \mu_1(\lambda, \rho_a, \rho_b)}$$

Recent developments

Conditioning on the current

Symmetric simple exclusion process with weak contacts

Infinite line

Conditioning on the current

Discrete time Markov process:

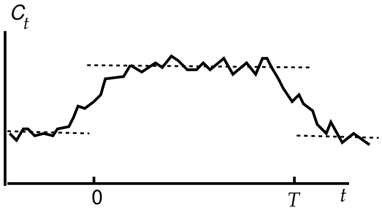
$M(C', C)$ probability of jumping from C to C' in one time step.

Trajectory $\{C_{-\infty}, \cdots C_t, C_{t+1}, \cdots\}$

$$\boxed{Q_T = \sum_{t=1}^{T} g(C_{t+1}, C_t)}$$

Main question:

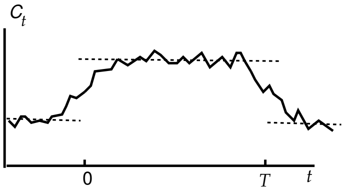
$$P(Q_T = Tq)\ ?$$



Large deviations in time

Chetrite Touchette 2015 ; D Sadhu 2019

$$Q_T = \sum_{t=1}^{T} g(C_{t+1}, C_t)$$



Questions: for a large time T

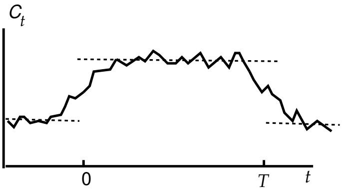
- Rate function $\phi(q)$

$$P(Q_T = Tq) \sim \exp\Big[-T\phi(q)\Big]$$

Large deviations in time

Chetrite Touchette 2015 ; D Sadhu 2019

$$Q_T = \sum_{t=1}^{T} g(C_{t+1}, C_t)$$



Questions: for a large time T

- Rate function $\phi(q)$

$$P(Q_T = Tq) \sim \exp\Big[-T\phi(q) \Big]$$

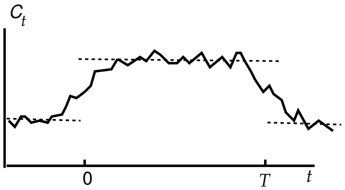
- Conditioned measure

$$P(C_t = C | Q_T = Tq)$$

Large deviations in time

Chetrite Touchette 2015 ; D Sadhu 2019

$$Q_T = \sum_{t=1}^{T} g(C_{t+1}, C_t)$$



Questions: for a large time T

- Rate function $\phi(q)$

$$P(Q_T = Tq) \sim \exp\left[- T\phi(q)\right]$$

- Conditioned measure

$$P(C_t = C | Q_T = Tq)$$

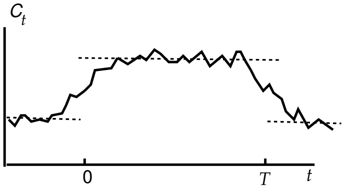
- Conditioned dynamics

$$P(C_{t+1} | C_t \text{ and } Q_T = Tq)$$

Large deviations in time

Chetrite Touchette 2015 ; D Sadhu 2019

$$Q_T = \sum_{t=1}^{T} g(C_{t+1}, C_t)$$



Questions: for a large time T

Rate function $\phi(q)$, conditioned measure or dynamics ?

Answer: use the deformed Markov matrix

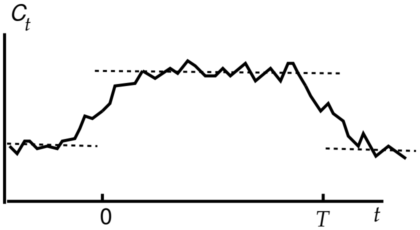
$$M(C', C) \longrightarrow M_\lambda(C', C) = M(C', C) \exp\left[\lambda\, g(C', C)\right]$$

If $e^{\mu(\lambda)}$ is the largest eigenvalue of M_λ

$$\mu(\lambda) = \max_q[\lambda q - \phi(q)]$$

The conditioned measure

$$M(C',C) \quad \longrightarrow \quad M_\lambda(C',C) = M(C',C)\exp\left[\lambda\Big(f(C)+g(C',C)\Big)\right]$$



$e^{\mu(\lambda)}$ eigenvalue of M_λ

$L_\lambda(C)$ and $R_\lambda(C)$ eigenvectors

- At $t=0$:

$$P(C_0 = C|Q_T) = R_0(C)\times L_\lambda(C)$$

- In the quasistationary regime ($t \gg 1$ and $T-t \gg 1$):

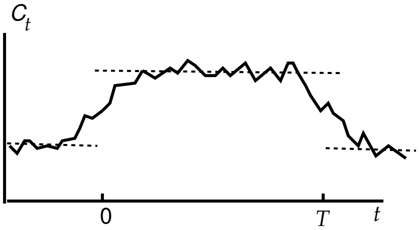
$$P(C_t = C|Q_T) = R_\lambda(C)\times L_\lambda(C)$$

- At $t=T$:

$$P(C_T = C|Q_T) = R_\lambda(C) = R_\lambda(C)\times L_0(C)$$

The conditioned dynamics

$$M(C', C) \quad \longrightarrow \quad M_\lambda(C', C) = M(C', C) \exp\left[\lambda g(C', C)\right]$$



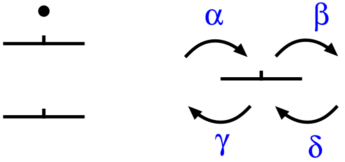
$e^{\mu(\lambda)}$ eigenvalue of M_λ

$L_\lambda(C)$ and $R_\lambda(C)$ eigenvectors

- In the quasistationary regime ($t \gg 1$ and $T - t \gg 1$):

$$\widetilde{M}(C', C) = \frac{L_\lambda(C')\, M_\lambda(C', C)}{e^{\mu(\lambda)}\, L_\lambda(C)}$$

A simple example: "a quantum dot" (continuous time)



$$M = \begin{pmatrix} -\beta - \gamma & \alpha + \delta \\ \beta + \gamma & -\alpha - \delta \end{pmatrix}$$

- $Q_T =$ time that the site is occupied

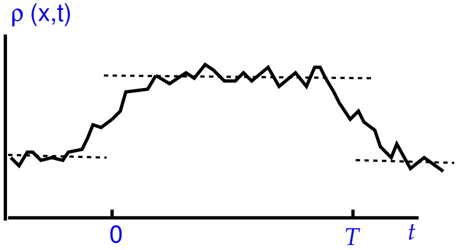
$$M_\lambda = \begin{pmatrix} \lambda - \beta - \gamma & \alpha + \delta \\ \beta + \gamma & -\alpha - \delta \end{pmatrix}$$

- $Q_T =$ integrated current

$$M_\lambda = \begin{pmatrix} -\beta - \gamma & \alpha\, e^{\lambda} + \delta \\ \beta + \gamma\, e^{-\lambda} & -\alpha - \delta \end{pmatrix}$$

NB : for the SSEP of L sites the size of the matrix is $2^L \times 2^L$

Conditioned measure and conditioned dynamics D Sadhu 2019



Quantities of interest

$$P\Big(\rho(x,T)\,|\,Q_T = q\,T;\rho(x,0)\Big)$$

$$P\Big(\rho(x,t)\,|\,Q_T = q\,T;\rho(x,0)\Big)$$

For large times T

$$P\Big(\rho(x,T)\,|\,Q_T = q\,T;\rho(x,0)\Big) \sim e^{-L\psi_{\text{left}}\left(\rho(x,0),q\right)-L\psi_{\text{right}}\left(\rho(x,T),q\right)-T\phi(q)}$$

The quasistionary regime

The conditioned measure

$$P(\{\rho(x)\}) \sim e^{-L\left[\psi_{\text{left}}\left(\rho(x),q\right)+\psi_{\text{right}}\left(\rho(x),q\right)\right]}$$

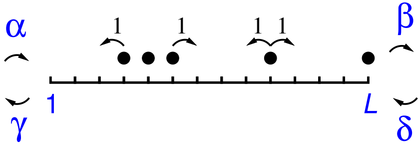
The conditioned dynamics

$$\frac{d\rho}{dt} = -\frac{dj}{dx}$$

and

$$j = -\frac{d\rho}{dx} + 2\rho(1-\rho)\,\frac{d}{dx}\left(\frac{\delta\psi_{\text{left}}}{\delta\rho(x)}\right)$$

SSEP Symmetric simple exclusion process

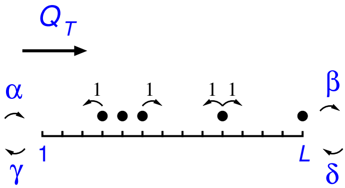


$$\rho_a = \frac{\alpha}{\alpha + \gamma} \qquad \rho_b = \frac{\delta}{\beta + \delta}$$

- Strong contacts $\alpha,\ \beta,\ \gamma,\ \delta\ = O(1)$ (normal case)
- Weak contacts $\alpha,\ \beta,\ \gamma,\ \delta\ = O\left(\frac{1}{L}\right)$

Baldasso, Menezes, Neumann, Souza 2017
Franco, Gonçalves, Neumann 2019
Gonçalves, Jara, Menezez, Neumann 2020
Bouley, Erignoux, Landim 2021

Large deviations of the current for strong contacts



Q_T = number of particles leaving the left reservoir during time T

$$\text{Pro}(Q_T = Tq) \sim e^{-T\,\phi(q)} \quad ; \quad \langle e^{\lambda Q_t} \rangle \sim e^{\mu(\lambda)\,T}$$

Strong contacts

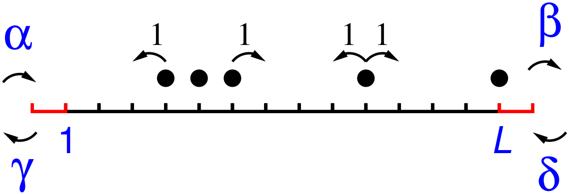
$$\mu(\lambda) = \frac{1}{L}[\log(\sqrt{1+\omega} + \sqrt{\omega})\,]^2$$

where $\omega = \rho_a(1-\rho_b)(e^{\lambda}-1) + \rho_b(1-\rho_a)(e^{-\lambda}-1)$

Bodineau D 2004

Bertini De Sole Gabrielli Jona-Lasinio Landim 2005

Large deviations of the current for weak contacts

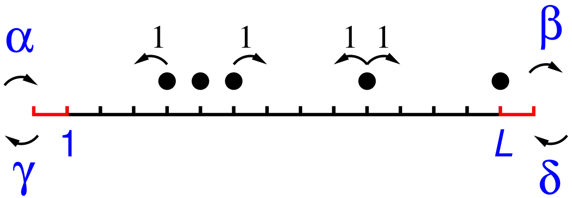


Strong contacts:

$\mu(\lambda)$ is known $\Rightarrow$ $\phi_{\text{bulk}}(q, \rho_a, \rho_b)$ is known

$$P_{\text{bulk}}(Q_T = Tq) \sim \exp[-T\,\phi_{\text{bulk}}(q, \rho_a, \rho_b)]$$

Large deviations of the current for weak contacts



Strong contacts:

$\mu(\lambda)$ is known $\Rightarrow$ $\phi_{\text{bulk}}(q, \rho_a, \rho_b)$ is known

$$\mathrm{P}_{\text{bulk}}(Q_T = Tq) \sim \exp[-T\,\phi_{\text{bulk}}(q, \rho_a, \rho_b)]$$

Weak contacts:

$$P(Q_T = Tq) =$$

$$\max_{\rho_1, \rho_2} \left[\mathrm{P}_{\text{left}}(Q_T, \rho_a, \rho_1) \times \mathrm{P}_{\text{bulk}}(Q_T, \rho_1, \rho_2) \times \mathrm{P}_{\text{right}}(Q_T, \rho_2, \rho_b) \right]$$

Large deviations of the current for weak contacts

$$\langle e^{\lambda\, Q_T} \rangle \sim e^{T\, \mu(\lambda)}$$

If

$$\omega = \rho_a(1-\rho_b)(e^{\lambda}-1) + \rho_b(1-\rho_a)(e^{-\lambda}-1) = \sinh^2 u$$

Strong contacts:

$$\mu(\lambda) = u^2$$

Weak contacts:

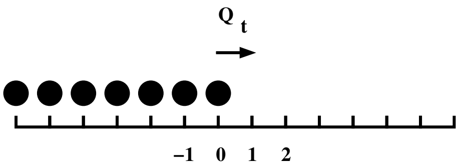
$$\mu(\lambda) = \frac{1}{L} \min_{t_a, t_b} \left[\frac{\sinh^2 t_a}{\Gamma_a} + (u + t_a + t_b)^2 + \frac{\sinh^2 t_b}{\Gamma_b} \right]$$

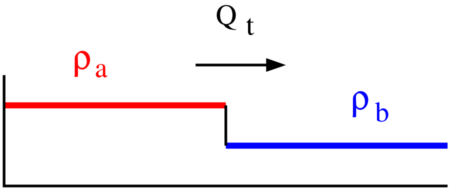
D Hirschberg Sadhu 2021

Remarks

1. One could add an arbitrary *finite* number of slow bonds in the bulk
2. The reasoning for the current should be valid for other diffusive systems
3. Picture different from the case of the large deviations of the density

Infinite line





ASEP

G. Schütz 98

⋮

Prähofer Spohn 2000-2002

⋮

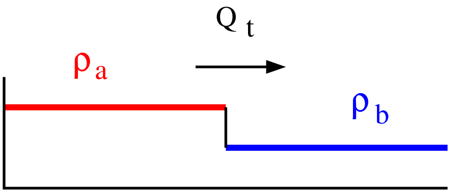
Tracy Widom 2008

$$\boxed{\langle e^{\lambda Q_t} \rangle = ?}$$

SSEP

D Gerschenfeld 2009

Infinite line



SSEP

D Gerschenfeld 2009

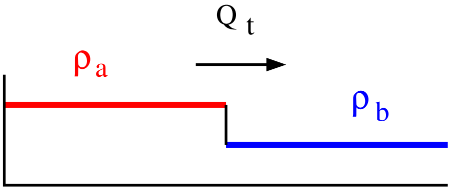
$$\boxed{\langle e^{\lambda Q_t} \rangle \simeq \exp[\sqrt{t}\ F(\omega)]}$$

where

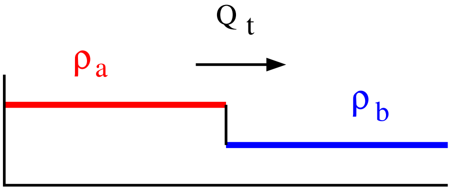
$$\boxed{\omega = \rho_a(e^{\lambda} - 1) + \rho_b(e^{-\lambda} - 1) + \rho_a\rho_b(e^{\lambda} - 1)(e^{-\lambda} - 1)}$$

and

$$\boxed{F(\omega) = \frac{1}{\pi} \int_{\infty}^{\infty} dk \log \left[1 + \omega e^{-k^2}\right]}$$



For large Q_t : $\boxed{\text{Pro}(Q_t) \sim \exp\left[-\frac{\pi^2}{12}\; Q_t^3/t\right]}$



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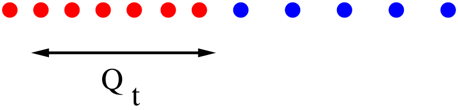
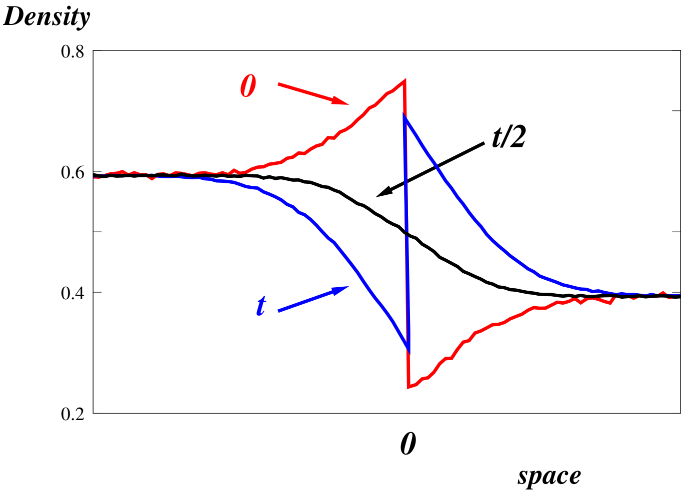
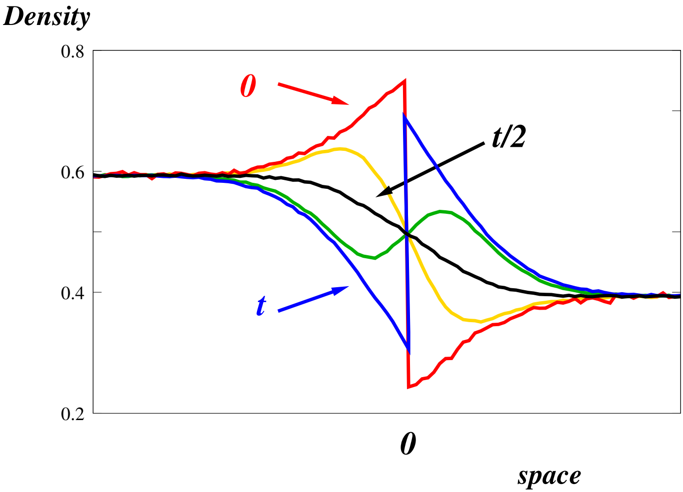


$$\mathrm{Pro}(Q_t) \sim \prod_{x=1}^{Q_t} \exp\left[-\frac{x^2}{t}\right]$$

Density profiles condionned on the current



Density profiles condionned on the current



Exact solutions using inverse scattering methods: 2021 →

- *Inverse Scattering Method Solves the Problem of Full Statistics of Nonstationary Heat Transfer in the Kipnis-Marchioro-Presutti Model* Phys. Rev. Lett. 2022
- *Full Statistics of Nonstationary Heat Transfer in the Kipnis-Marchioro-Presutti Model* arXiv:2204.06278

Bettelheim, Smith, Meerson

- *Exact solution of the macroscopic fluctuation theory for the symmetric exclusion process* arXiv:2202.05213.

Mallick, Moriya, Sasamoto

- *The crossover from the Macroscopic Fluctuation Theory to the Kardar-Parisi-Zhang equation controls the large deviations beyond Einstein's diffusion* arXiv:2204.04720

Krajenbrink, Le Doussal

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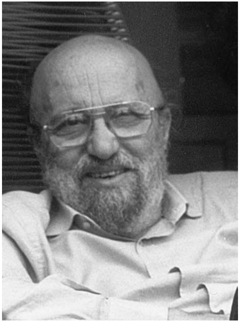

Gianni Jona-Lasinio.

Preface

It's a great pleasure to dedicate this issue of the *Journal of Statistical Physics* to Gianni Jona-Lasinio, on the occasion of his seventieth birthday.

As is clear from the articles submitted to this issue, Gianni is universally regarded with great respect and affection both for his contributions to science and for the way he conducts himself.

I consider myself fortunate to have known Gianni for a long time and to have enjoyed his friendship and wisdom, both in scientific and other matters. I am sure I speak for the whole statistical mechanics and scientific community in wishing Gianni many many happy, productive, and healthy years.

Joel L. Lebowitz

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