Mini-corso di dottorato

"Counting and dynamics in negative curvature"

by Marc Peigné

Il corso (15 ore) inizierà LUNEDI' 10 OTTOBRE, 15-17 aula B, e proseguirà con cadenza bisettimanale (lunedì/mercoledì ore 15-17 circa) fino a metà novembre.

Tentative program

In this course, we aim to explain some relations between some classical results on the dynamics of the geodesic flow of a negatively curved manifold X_0 (namely: a quotient $X_0 = X \backslash G$ of a Cartan-Hadamard space X, with sectional curvature $-b^2 \leq k(X) \leq -a^2 < 0$, by a discrete group of isometries G) and the asymptotic behavior of some classical counting functions associated to G.

In the first part of the course, we will give the necessary background. In particular, we will recall the basic notions of: geometry of a Cartan-Hadamard space X, the ideal boundary ∂X and Busemann functions, discrete Kleinian groups G and the limit set LG, the representation of the geodesic flow of $X_0 = G \backslash X$; we will then introduce more refined tools such as the Patterson-Sullivan δ-conformal densities $\mu = (\mu_x)_{x \in X}$ (a family of measures on the ideal boundary, indexed by points of X, satisfying some quasi-invariance property under the action of G) and the Bowen-Margulis measure on the unit tangent bundle SX (which is both invariant under the action of G and of the geodesic flow).

This part will be preparatory for a number of deep results of dynamics in negative curvature, which will be the object of the second part of the course. We will mainly focus on the class of geometrically finite manifolds, which contains in particular all negatively curved manifolds of finite volume, and we aim to prove:

- the Hopf-Tsuji-Sullivan theorem, giving equivalent conditions to the conservativity and ergodicity of the geodesic flow on SX_0, with respect to the Bowen-Margulis-Sullivan measure m;

- the relation between finiteness of the Bowen-Margulis measure and convergence/divergence of some Poincaé series associated to parabolic subgroups;

- estimates of the orbital function $N_G(x, y, R)$ (the number of orbit points Gy in X, falling in a ball of center x and radius R);

- the relation between Hausdorff and Patterson-Sullivan measures on ∂X, and between the critical exponent of G and the dimension of its limit set.