

Dottorato in Matematica

Contour integral methods for evolutionary PDEs

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Topics

I. Motivation.

Linear and semilinear parabolic problems. PDEs of fractional order in time. PDEs with memory terms of convolution type. Transparent boundary conditions for Schrödinger equations. Exterior wave problems.

II. **Vector-valued Laplace transforms and Cauchy problems.** The Bochner Integral. Convolutions. Existence of the Laplace integral. Analytic behaviour. Operational properties. Laplace transforms of Exponentially Bounded Functions. Laplace transforms of Holomorphic Functions. C_0 -semigroups and Cauchy Problems. Holomorphic Semigroups.

III. **The inverse Laplace transform.** The Bromwich inversion formula. Trapezoidal quadrature for analytic integrands. The numerical inversion of sectorial Laplace transforms.

IV. **Convolution Quadrature (CQ).** Discrete convolutions with CQ. The ζ transform. CQ based on BDF formulas and Runge–Kutta methods. Operational properties of CQ. Stability and convergence. Application of CQ to the boundary integral formulation of the wave equation.

References

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