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THE SERIES IS INDEXED IN SCOPUS

Emiliano Cristiani • Maurizio Falcone •
Silvia Tozza
Editors

Mathematical Methods for Objects Reconstruction

From 3D Vision to 3D Printing

Springer

Editors
Emiliano Cristiani
Istituto per le Applicazioni del Calcolo "M.Picone"
Consiglio Nazionale delle Ricerche
Rome, Italy

Maurizio Falcone *(deceased)*
Department of Mathematics
Sapienza University of Rome
Rome, Italy

Silvia Tozza
Department of Mathematics
University of Bologna
Bologna, Italy

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To Maurizio,
our Academic Father

Preface

Three-dimensional (3D) reconstruction of the shape of objects is an issue that has been investigated largely by the computer vision and applied mathematics communities since the last century. The class of problems related to that issue is the so-called Shape-from-X class, where the X specifies the kind of data used for the reconstruction (e.g., shading, texture, template, polarization).

The main Shape-from-X techniques can be classified as photometric or geometric based, and/or in relation to the number of images they require. Geometric Shape-from-X techniques are built upon the identification of features in the image(s). An example is the Shape-from-Template (SfT), which uses a shape template to infer the shape of a deformable object observed in an image. On the other hand, photometric Shape-from-X techniques are based on the analysis of the quantity of light received in each photosite of the camera's sensor. An example can be the classical Shape-from-Shading problem, formalized in that sense since the pioneering work by B.K.P. Horn in the 1970s, which utilizes as a datum a single 2D gray-level image of the object to be reconstructed.

All these 3D reconstruction problems are typically ill posed since they do not admit, in general, a unique solution. Hence, advanced techniques for the analysis and for the numerical approximation and/or a priori knowledge are required.

Most of the problems belonging to the Shape-from-X class can be formulated via partial differential equations and/or via variational methods, giving rise to a variety of nonlinear systems that have been analyzed by many authors.

Shape-from-X problems have a natural counterpart in 3D printing, which consists in producing a 3D printed object with desired appearance and physical properties. Nevertheless, the two areas have received different attention from the computer vision and applied mathematics communities, Shape-from-X being much more investigated than the other. The mathematical aspects of 3D printing have begun to be explored only since 2017. So far, research has focused mainly on the shape optimization of overhangs, those temporary structures that must be printed to support the actual object during construction and then removed at the end of the process. To this end, long-standing and mature mathematical tools for shape optimization have been adapted to this new application, leading to great results (especially for saving printing material). In particular, we are referring to those mathematical tools based on elastic displacement, Hamilton-Jacobi equations, front propagation problems, and level set method.

Only recently, mathematical research has expanded somewhat in order to include optimal path generation of the extruder (important for reducing printing time), minimization of thermal stress (important for the quality of the final product), and optimal object partitioning.

One of the most interesting problems that fully relates 3D vision to 3D printing is probably the appearance replication. This problem, only partially explored, consists in replicating (multi-material) real objects with complex reflectance features using a single, cheaper printing material, possibly with the simple diffuse Lambertian reflectance. To trick the eye, the surface of the object is rippled with tiny facets that regulate the reflection of light, analogous to what is done, for example, in the Oren-Nayar reflectance model for recovering the 3D shape of the object in the context of the Shape-from-Shading problem.

This volume is devoted to mathematical and numerical methods for object reconstruction, and it aims at creating a bridge between 3D vision and 3D printing, moving from the 3D data acquisition and 3D reconstruction to the 3D printing of the reconstructed object, with software development and/or new mathematical methods to get closer and closer to real-world applications. Some contributions focus on 3D vision, dedicated to photometric- or geometric-based Shape-from-X problems. Other contributions address specific issues related to 3D printing, further widening the research topics of this newly investigated area.

This book is useful for both academic researchers and experts from industry working in these areas who want to focus on complementary knowledge in 3D vision and 3D printing fields. Also practitioners and graduate students working on partial differential equations, optimization methods, and related numerical analysis will find this volume interesting as an approach to the field.

The research contributions contained in the book give only a partial overview of the research directions and various techniques of heterogeneous origin discussed during the INdAM Workshop "Mathematical Methods for Objects Reconstruction: From 3D Vision to 3D Printing", held online due to COVID-19 pandemic, February 10–12, 2021.

We want to thank all the speakers at that workshop and those who contributed to the present volume, which we hope will attract new researchers to this challenging area.

A particular mention goes to Maurizio Falcone, our Academic Father, who disappeared suddenly and prematurely in November 2022, during the publication process of this book. Maurizio followed carefully almost all the steps of this book, as co-editor and also as co-author of the first Chapter. We thank him for what he did for us and in general for his contributions to Applied Mathematics. We miss him, a feeling shared by all the authors of this book, colleagues and friends. He leaves an unfillable void in our lives.

The following is a brief description of the chapters contained here.

In Chap. 1, Cristiani et al. present a brief overview of techniques for 3D reconstruction (solving the classical Shape-from-Shading, Photometric Stereo, and Multi-view Shape-from-Shading problems), and some issues for 3D printing, for example, overhang and infill, emphasizing the approaches based on nonlinear partial differential equations and their numerical resolution. The goal is to present introductory materials to readers, stressing the common mathematical techniques and the possible interactions among the two areas, which nowadays are still limited. The chapter also presents two appendices on STL and G-code file formats, which are largely used in 3D printing and are useful to create a final 3D print from scratch without relying on existing compiled software.

In Chap. 2, Rodriguez et al. focus on uncalibrated photometric-stereo problem using non-Lambertian reflectance models. In more detail, Hayakawa's procedure is used to detect light positions, provided that at least six images of the sample object with different lighting directions are available, and the Oren-Nayar model is used as preprocessing for tuning Hayakawa's detected light directions. The authors stress the importance of the roughness parameter in estimating the light directions and in reconstructing the object. They determine a realistic range of variation for the roughness parameter, which results in a set of meaningful 3D reconstructions in an outdoor environment.

In Chap. 3, Collins and Bartoli tackle the SfT problem, aiming to reconstruct the 3D shape of a deformable object from a monocular image. The authors present a novel SfT method that handles unknown focal length, calling it fSfT. The fSfT problem is solved by gradient-based optimization of a large-scale nonconvex cost function, which requires a suitable initialization, and cost-normalization strategies are presented, allowing the same cost-function weights to be used in a diverse range of cases. Numerical results on 12 public datasets are reported in order to show the effectiveness of the proposed fSfT method in both focal length and deformation accuracy for real-world applications.

In Chap. 4, Bærentzen et al. focus on 3D reconstruction from a discrete set of data points in space, that is, a point cloud. Specifically, they are interested in 3D reconstruction of real-world objects, with very thin tubular structures, which are hard to reconstruct using traditional methods. The proposed procedure constructs a skeleton of the object from a graph, whose vertices are the input points. Then, a surface representation is created from the skeleton, and, finally, a triangular mesh is generated from the surface representation. Following this pipeline, they are able to reconstruct a valid surface model from the data. The results demonstrate the efficacy of the proposed method on a tree acquired using ground-based LiDAR.

In Chap. 5, Beisegel et al. identify a novel optimization problem in both wire arc additive manufacturing and powder bed fusion 3D printing processes. The problem belongs to the class of mixed integer programming techniques with partial differential equations as a further constraint. An important novelty here is that heat transfer is taken into account in the optimization process, aiming at lowering the internal thermal stress of the object.

In Chap. 6, Yarahmadi et al. deal with the powder bed fusion printing process and propose an optimization heuristic to find the optimal laser beam trajectory. The devised optimization procedure, as in the previous chapter, takes into consideration the thermal stress of the object in order to minimize the average thermal gradient.

Rome, Italy Emiliano Cristiani
Bologna, Italy Silvia Tozza
December 9, 2022

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