

# Maurizio day

## Numerics for controls, pdes and images

Rome, December 15, 2023

### 1 Adriano Festa

**Title:** Forward-Forward Mean Field Games in mathematical modeling with application to opinion formation and voting models.

**Abstract:** While the general theory for the terminal-initial value problem in mean-field games is widely used in many models of applied mathematics, the modeling potential of the corresponding forward-forward version is still under-considered. In this presentation, we show the well-posedness of the problem in a quite general setting and explain how it is appropriate to model a system of players that have a complete knowledge of the past states of the system and are adapting to new information without any knowledge about the future. Then, we show how forward- forward mean field games can be effectively used in mathematical models for opinion formation and other social phenomena.

### 2 Athena Picarelli

**Title:** A deep solver for BSDEs with jumps

**Abstract:** The aim of this work is to propose an extension of the deep solver by Han, Jentzen, E (2018) to the case of forward backward stochastic differential equations (FBSDEs) with jumps. As in the aforementioned solver, starting from a discretized version of the FBSDE and parametrizing the (high dimensional) control processes by means of a family of artificial neural networks (ANNs), the FBSDE is viewed as model-based reinforcement learning problem and the ANN parameters are fitted so as to minimize a prescribed loss function. We take into account both finite and infinite jumpactivity by introducing, in the latter case, an approximation with finitely many jumps of the forward process. We successfully apply our algorithm to option pricing problems in low and high dimension and discuss the applicability in the context of counterparty credit risk.

### 3 Alessio Oliviero

**Title:** Optimal control of epidemiological models

**Abstract:** The study of compartmental models has been largely developed in literature to examine or simulate the evolution of an epidemic, especially in the latest years. We apply an optimal control approach to this kind of models, considering two main ingredients to contain the spread of an infectious disease: vaccination and restrictive measures. We begin with a variation of the classic SEIR model, showing that even in rather simple scenarios the cost functional can present multiple local minima, therefore the widely used variational approach (which exploits Pontryagin's principle) can easily lead to sub-optimal solutions. In order to obtain reliable controls, we propose a "correction" of the previous approach using Dynamic Programming. Finally, we present an on-going collaboration with the Istituto Superiore di Sanità, whose aim is to use the optimal control framework on a large-scale model for measles, built and calibrated on real data. This is a joint work with Simone Cacace (Sapienza University of Rome).

### 4 Silvia Tozza

**Title:** A scheme for the game  $p$ -Laplacian and its application to image inpainting

**Abstract:** We propose a new numerical scheme for the game  $p$ -Laplacian, based on a semi-Lagrangian approximation. We focus on the 2D version of the game  $p$ -Laplacian, with the aim to apply the new scheme in the context of image processing. Specifically, we want to solve the so-called inpainting problem, which consists in reconstructing one or more missing parts of an image using information taken from the known part. The numerical tests show the reliability of the proposed method and the advantages of taking a  $p > 1$  in terms of execution time and accuracy. Joint work with Elisabetta Carlini.

### 5 Alessandro Alla

**Title:** A POD based approach to identify and control PDEs

**Abstract:** This talk focuses on presenting an algorithm designed to efficiently identify and control unknown two-dimensional Partial Differential Equations (PDEs). We consider our system as a black box that takes the control as input. The control strategy is based on the State-Dependent Riccati approach, whereas the identification of the model on Bayesian linear regression. To enhance computational efficiency, we employ Proper Orthogonal Decomposition (POD), leveraging orthogonal projections to reduce the dimensionality of the problem.

At each iteration, based on the observed data, we derive an estimate of the a-priori unknown parameter configuration of the (reduced) PDE and subsequently compute the control for the corresponding model. The talk will feature numerical examples to illustrate the accuracy and effectiveness of our method. This is a work in progress with Agnese Pacifico.

## 6 Francesca Ignoto

**Title:** Dissoluzione di particelle di farmaco di forma variabile con il metodo Level-Set

**Abstract:** La dinamica della dissoluzione di più particelle di forma variabile è fondamentale per la previsione della dissoluzione di farmaci in un fluido. In questo talk accenno ad un lavoro che mira ad estendere, attraverso il metodo level-set, un precedente lavoro del 2022 [1]. In quest'ultimo è stato sviluppato un modello matematico per descrivere il processo di dissoluzione di una particella di farmaco considerando il suo fronte come un insieme discreto di punti. Il nostro lavoro, invece, sfrutta la rappresentazione del fronte della particella come curva di livello zero della funzione level-set. I vantaggi derivanti dall'utilizzo di questo metodo permettono di considerare facilmente particelle di forma rettangolare e di simulare la dissoluzione di una distribuzione di particelle di forme e dimensioni diverse.

[1] M. Abrami, M. Grassi, D.Masiello, G. Pontrelli, Dissolution of irregularly-shaped drug particles: mathematical modelling, *European Journal of Pharmaceutics and Biopharmaceutics*, 177, 199-210, 2022.

## 7 Giulia Villani

**Title:** Optimal control for orbital transfer of LEO satellites with Low-Thrust engines

**Abstract:** The research project, in collaboration with Thales Alenia Space Italia SpA, aims to study, develop and numerically simulate innovative methods for optimizing the orbital transfer of small satellites in LEO (Low Earth Orbit) using Low-Thrust engines, to be deployed in constellations for Earth observation applications. We developed the controlled dynamics of a single satellite, using the Dynamic Programming approach, based on the characterization of the value function via the Hamilton-Jacobi-Bellman equation. We have built an algorithm that fits a specific physical problem of industrial interest, applying numerical techniques as the Policy Iteration to make the algorithm faster, and using a more suitable grid to save memory. The main challenge is to build a solid control model to satisfy mission objectives and requirements, e.g. the time needed to reach the target orbit, or the use of propellant.