Numerical methods for PDEs: optimal control, games and image processing

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Abstracts

Yves Achdou (Université Paris-Diderot, France)

Two-domains Hamilton Jacobi equations with an oscillating interface: effective transmission conditions

Abstract: We study regional optimal control problems in R^d. We assume that the space is divided into two unbounded subdomains. The interface is the graph of a smooth periodic real value function. The period and the amplitude of the interface oscillations are of the order of ϵ, where ϵ is a small parameter that will tend to zero. The dynamics and the running costs are discontinuous across the interface. The value function is the solution of an Hamilton-Jacobi equation with the transmission conditions proposed by S. Oudet [2] or Imbert-Monneau [3].
We study the homogenization of the problem. We prove that at the limit when ϵ goes to 0, the value function tends to the value function of an effective control problem in two half-spaces. The effective transmission on the flat interface is found by solving a sequence of state constraint problems in larger and larger bounded domains.
We use the techniques that were introduced in [1, 4].

Bibliografy:

[1] Y. Achdou, N. Tchou Hamilton-Jacobi equations on networks as limits of singularly perturbed problems in optimal control: dimension reduction, Comm. PDE, (2014),
[2] S. Oudet Hamilton-Jacobi equations for optimal control on multidimensional junctions preprint
[3] C. Imbert, R. Monneau Quasi-convex Hamilton-Jacobi equations posed on junctions: the multi-dimensional case. arXiv preprint arXiv:1410.3056 (2014).
[4] G. Galise, C. Imbert, R. Monneau A junction condition by specied homogenization. arXiv preprint arXiv:1406.5283 (2014).

Alessandro Alla (University of Hamburg, Germany)

On the coupling between MPC and DP methods for optimal control problems

Abstract: We consider the approximation of an infinite horizon optimal control problem which combines a first step based on Model Predictive Control (MPC) in order to have a quick guess of the optimal trajectory and a second step where we solve the Bellman equation in a neighborhood of the reference trajectory.
The direct global solution approach via the Bellman equation can be rather expensive due to the fact that we have to set the problem in a domain containing all the possible initial conditions x for the dynamics. On the other hand, the main feature of MPC is to compute an approximate feedback control for the dynamics starting at a given initial condition x by solving a sequence of finite horizon optimal control problems. Therefore, we first solve the problem for a given initial condition via MPC and then compute the value function in a neighborhood of that trajectory. The second step is necessary to allow for a more stable solution where we can use the information around the reference trajectory (and not only those on the reference trajectory).
We will present the main features of these new technique, and illustrate some numerical tests in order to show the effectiveness of this coupling. Work in collaboration with M. Falcone and G. Fabrini.

Martino Bardi (Università di Padova, Italy)
A Dijkstra-type algorithm for dynamic games

Abstract: We study zero-sum dynamic games with deterministic transitions and alternating moves of the players. Player 1 aims at reaching a terminal set and minimising a running and final cost. We propose and analyse an algorithm that computes the value function of these games extending Dijkstra's algorithm for shortest paths on graphs. We also show the connection of these games with numerical schemes for differential games of pursuit-evasion type, if the grid is adapted to the dynamical system. Under suitable conditions we prove the convergence of the value of the discrete game to the value of the dfferential game as the step of approximation tends to zero. Joint work with Juan Pablo Maldonado Lopez.

Isabeau Birindelli (Sapienza Università di Roma, Italy)
A variational approach to fully nonlinear operators

Abstract: TBA

Olivier Bokanowski (Université Paris-Diderot, France)
A high order numerical method for a nonlinear diffusion + obstacle equation

Abstract: The talk is concerned with high order schemes for Hamilton-Jacobi equations. We will focus on a new attempt for getting a provably second order scheme for the american option problem, which is a kind of diffusion equation with an obstacle term (joint work with K. Debrabant).

Luca Bonaventura (Politecnico di Milano, Italy)
A domain decomposition approach to exponential time integration of PDEs

Abstract: Exponential methods are considered as an alternative to semi-Lagrangian methods for advection dominated flows in environmental fluid dynamics problems. The accuracy and performance of some exponential methods is assessed by comparison with these more common time discretization techniques. In order to increase the efficiency of the standard exponential methods presented in the ODE literature, a domain decomposition approach to the computation of matrix exponentials will also be discussed, that allows to achieve greater parallel efficiency and to decrease the computational cost of exponential methods.

Fabio Camilli (Sapienza Università di Roma, Italy)
Eikonal equations on the Sierpinski gasket

Abstract: We introduce a definition of eikonal equation on the Sierpinski gasket in the spirit of the construction of the Laplacian on fractals in Kigami (Analysis on fractals, Cambridge University Press, 2001): we consider graph eikonal equations on pre- fractals and we show that the solutions of these problems converge to a function efined on the Sierpinski gasket as the prefractals approach this set. Moreover we characterize this limit function as the unique metric viscosity solution to the eikonal equation on the Sierpinski gasket according to the definition introduced in Y. Giga, N. Hamamuki, A. Nakayasu (Eikonal equations in metric spaces, Trans AMS, to appear).

Italo Capuzzo Dolcetta (Sapienza Università di Roma, Italy)
Semidiscrete and finite differences approximation of mean field games

Abstract: I will take the move from some old work of mine on approximation of Hamilton-Jacobi equations to introduce a recent implementation due to Camilli and Silva of those ideas to the more general setting of deterministic mean field games.
The final part of the talk will briefly report on joint work with Achdou and Camilli on finite differences schemes for second order mean field games type systems.

Jean-Denis Durou (IRIT, Toulouse, France)
Photometric stereo: A photographic technique for 3D-scanning

Abstract: Shape-from-shading is a well-known, although ill-posed, photographic technique for 3D-reconstruction. Photometric stereo is an extension of shape-from-shading, where several light sources are used to illuminate from different directions the scene to be reconstructed. This problem is well-posed when the light sources are known. I will first recall the main features of photometric stereo, including the case where the light sources are unknown (uncalibrated photometric stereo). Then I will focus on some recent applications to 3D-scanning which have been recently developed in our laboratory.

Roberto Ferretti (Università Roma Tre, Italy)
Some issues in the semi-Lagrangian treatment of second-order balance laws

Abstract: While the original idea of Semi-Lagrangian (SL) schemes typically applies to first-order evolutive equations in advective form, the SL framework has been later adapted to more general situations. Second-order problems have been treated by mixing SL with stochastic techniques, and Flux-Form SL (FFSL) schemes have been proposed to treat equations in the form of balance laws. Recently, these two extensions have been further mixed to tackle second-order linear continuity equations.
With the final goal of nonlinear models in mind, we will address in this talk some recent results and ongoing works on the topic, focusing in particular on the properties of conservation and entropicity for SL and FFSL schemes.

Adriano Festa (UMA ENSTA-ParisTech, Palaiseau, France)
Independent sub-domains reconstruction and parallel computing

Abstract: In this talk I'll present a numerical technique for the parallel computing of a solution of an Hamilton-Jacobi equation. The convergence of the algorithm will be proved introducing the concept of independent sub-domain; point that will permit an easy analysis of the features of the procedure. I'll discuss some special aspects and some possible developments of the technique.

Stefano Finzi Vita (Sapienza Università di Roma, Italy)
Similarity solutions for the silo filling problem in granular matter theory: a numerical study

Abstract: We give a numerical characterization of similarity quasi-static solutions for the so-called silo problem described by the two-layer model of Hadeler and Kuttler. Such solutions arise when an empty silo is filled by a constant in time source of granular matter: in finite time the free surface evolves towards a well-defined profile which then keeps its shape growing with a fixed velocity. Explicit integral expressions for it are known only for the one dimensional or the radial cases. In the general situation it can be approximated by the numerical solution of a stationary indefinite Neumann problem. We also present a finite difference scheme for the dynamical model which shows convergence in time of the discrete solutions towards the previously characterized similarity solutions.

Tiziana Giorgi (New Mexico State University)
Field-induced smectic phases in liquid crystals

Abstract: We will discuss a phenomenological Landau energy introduced by Vaupotič and Čopič to study the onset of layer-undulated structures, induced by an applied electric field, in bent-core molecules liquid crystals. We will also present an analysis of the chevron structure in Smectic A liquid crystals due to magnetic fields, as modeled by a Chen-Lubensky energy. The presentation will refer to joint work with Carlos-García Cervera and Sookyung Joo.

Lars Grüne (University of Bayreuth, Germany)
Stabilization with discounted optimal control

Abstract: It is known that undiscounted infinite horizon optimal control can serve as an auxiliary problem for computing stabilizing controls for nonlinear systems. Unfortunately, however, these kind of optimal control problems are often very difficult to solve numerically. In this talk we show that under suitable conditions discounted infinite horizon optimal control can also be used for stabilization. We discuss the needed conditions and compare them with alternative stabilization approaches in the literature. Moreover, we present a numerical example illustrating the method. The talk is based on joint work with Vladimir Gaitsgory (Sydney) and Neil Thatcher (Adelaide).

Nicola Guglielmi (Università dell'Aquila, Italy)
Most unstable switching laws for switched linear systems

Abstract: When considering the stability under arbitrary switching of a discrete-time linear switched system

$$x(k+1) = A_{\sigma(k)}\, x(k), \quad \sigma : N \to \{1, \dots, m\}, \quad A_1, \dots, A_m \text{ given matrices},$$

one is typically interested in determining the most unstable switching law (MUSL).
This is equivalent to computing the so-called joint spectral radius of the underlying set of matrices. If the solution of the switched system corresponding to the MUSL converges to zero, then the switched system is stable for any switching law. It is well- known that the MUSL can be characterized using optimal control techniques. Such variational approach leads to a Hamilton–Jacobi–Bellman equation describing the behavior of the switched system under the MUSL. The solution of this equation is sometimes referred to as a Barabanov norm (N.E. Barabanov, 1988) of the switched system. Although the Barabanov norms have been studied extensively, even in very simple cases it is very difficult to construct them and determine the shape of their unit balls (see, e.g., Kozyakin, 2010)). The reasons of this is that they are not defined in a constructive way.
In this talk we present a methodology to construct polytope Barabanov norms, i.e. we give a canonical procedure, exploiting recent methods (described in Guglielmi, Wirth and Zennaro, 2005 and Guglielmi and Protasov, 2013), which allows to determine generically a polytope Barabanov norm.
This is a joint work with Marino Zennaro (Università di Trieste).

Dante Kalise (RICAM, Austria)
Local optimization in semi-Lagrangian schemes for Hamilton-Jacobi equations

Abstract: In this talk we introduce local optimization strategies for Dynamic Programming-based approximations of Hamilton-Jacobi equations. In particular, semi-Lagrangian schemes require the local minimization of the Hamiltonian with respect to the control variable. For this purpose, we set a semi-smooth Newton method and a first-order primal-dual algorithm, both leading to accurate optimal control fields. We present different examples assessing the performance and accuracy of the proposed scheme.

Roberto Mecca (Istituto Italiano di Tecnologia, Italy)
A Different(ial) Approach to the Photometric Stereo Problem

Abstract: Photometric Stereo (PS) is a fundamental problem in Computer Vision aimed at reconstructing surface depth given multiple images under different illuminations. It can be expressed as a non-linear system of Hamilton Jacobi equations in surface derivatives when 3 or more images are given. We talk about realistic PS modeled by PDEs starting from the problem when only 2 images are available. We extend our previous results on this problem, considering the more realistic perspective projection of surfaces during the photographic process and point light source parametrization. We prove that the new differential formulation is well-posed leading to quasi-linear PDEs.

Athena Picarelli (ENSTA ParisTech & INRIA Saclay-Ile de France, France)
State-constrained stochastic optimal control problems via reachability approach

Abstract: This work is concerned with a general class of stochastic optimal control problems in presence of state-constraints. When state-constraints are taken into account and in absence of quite restrictive controllability assumptions on the dynamics, the continuity of the value function cannot be guaranteed and some well-known problems arise in its characterization as a viscosity solution of a Hamilton-Jacobi-Bellman equation. The approach proposed in this work leads to a characterization of the epigraph of the value function translating, at a first stage, the optimal control problem into a state-constrained stochastic target problem with unbounded controls. This new formulation of the problem has the advantage to allow to solve it by a level set approach, where the state-constraints can be managed by an exact penalization technique.

Franco Rampazzo (Università di Padova, Italy)
Costly asymptotic controllability (Si ad metam gratuitus non est accesus)

Abstract: As is well-known, the existence of a Control Lyapunov Function guarantees global asymptotic controllability. More generally, if a positive integral cost functional is given besides the dynamics, a *Minimum Restraint Function* (shortly, MRF) is a particular Lyapunov Function which verifies a stronger Hamilton-Jacobi differential inequality and, as a counterpart, provides a bound on the cost one pays to approach the target. (In terms of PDE's this might be read as a comparison result together with information on (quasi-) characteristics.)
I will address the question of the MRF method for systems with *unbounded controls*. I will also mention some algebraic results which render the search for a MRF easier in the case when the dynamics is polynomial in the control variable.
Most of the talk's contents will be based on joint work with Anna Chiara Lai.

Giovanni Russo (Università di Catania, Italy)
Semilagrangian schemes for kinetic equations

Abstract: New semilagrangian methods for two types of kinetic equations will be considered, namely BGK model of rarefied gas dynamics and Vlasov-Poisson equation. For the BGK model high order semilagrangian schemes are developed which are able to treat the equation for a wide range of Knudsen number. High order in time is obtain by high order RK or BDF schemes, while high order in space is obtained by WENO reconstruction. Several issues will be discussed, such as extension of the approach to the BGK systems for mixtures, as well as the treatment of reflective and diffusive boundary conditions. The second problem we consider is the development of semilagrangian schemes for the Vlasov-Poisson system. The equation is integrated backward along the characteristics, and the function is reconstructed at the foot of the characteristics by WENO in several dimensions. A conservative version of the scheme will be presented, which

unfortunately suffers from severe stability restrictions. A proper stability analysis of such conservative semilagrangian scheme is still an open problem.

Smita Sahu (Sapienza Università di Roma, Italy)
An efficient fitered scheme for some first order Hamilton-Jacobi Bellman equations

Abstract: We introduce a new class of “Filtered” schemes for some first order non-linear Hamilton-Jacobi-Bellman equations. The work follows recent ideas of Froese and Oberman (SIAM J. Numer. Anal., Vol 51, pp.423-444, 2013). The proposed schemes are not monotone but still satisfy some ϵ-monotone property. Convergence results and precise error estimates are given, of the order of $\sqrt{\Delta x}$ where Δx is the mesh size. The framework allows to construct finite difference discretizations that are easy to implement, high order where the solution is smooth, and provably convergent, together with error estimates. Numerical tests on several academic examples are given to validate the approach, also showing how the fitered approach can stabilize an otherwise unstable high-order scheme.
Joint work with Olivier Bokanowski, and Maurizio Falcone.

Antonio Siconolfi (Sapienza Università di Roma, Italy)
Weak KAM techniques for singularly perturbed control problems

Abstract: Even if the first connections between Weak KAM and viscosity solutions theory has been established in the framework of asymptotic problems as homogenization of Hamilton-Jacobi PDE, it is, however, a matter of fact that the specic impact of weak KAM technology for such kind of models has been so far irrelevant.
The novelty of the singularly perturbed optimal control problems, with slow and fast variables, we present, is instead that an accurate preliminary Weak KAM analysis is needed in order to construct correctors and hereafter apply Evans' perturbed test function method to pass to the limit in the fast variable. More specifically, we approximate the Hamiltonians of the original problems by coercive ones on which Weak KAM tools are applied. The peculiarity of the setting, requiring the aforementioned surplus of qualitative analysis, is the noncompactness of the fast variable. This is somehow counterbalanced by coercivity of the running cost; a suitable controllability condition is also assumed, as customary for these problems.

Silvia Tozza (Sapienza Università di Roma, Italy)
A unified approach to Shape-from-Shading models for non-Lambertian surfaces

Abstract: In this talk we will present two Shape-from-Shading models for non-Lambertian surfaces and compare them with the classical Lambertian model in a unified framework. These non-Lambertian models have been proposed in the literature by various authors in previous papers in order to take into account more realistic surfaces such as rough and specular surfaces.
The advantage of our unified mathematical formulation is the possibility to easily adapt a singledifferential model to different situations only by changing some parameters.
In addition, the numerical semi-Lagrangian approximation we propose is valid for the general model and can be adapted in a rather easy way. We will compare the models on some benchmark images coming from real and virtual images.

Hasnaa Zidani (ENSTA ParisTech, France)
Optimal control problems on generalized networks

Abstract: This talk concerns some optimal control problems on d-dimensional networks. We will present a new characterization of the value function as solution to a system of Hamilton- Jacobi-Bellman equations with appropriate junction conditions. This result doesn't require any condition on the junctions and still holds when the value function is discontinuous.