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(joint work with Gilles Aubert and Laure Blanc-Féraud)

ADMM algorithm for demosaicking of color images

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Metodi matematici nel trattamento delle immagini
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Outline

Bayer Filter

Initial color image in the RGB basis

$$(i,j) \in Z^2 \mapsto u^c = \{u(i,j)\}_{(i,j)\in Z^2} \quad u^c = (u^R, u^G, u^B)^T$$

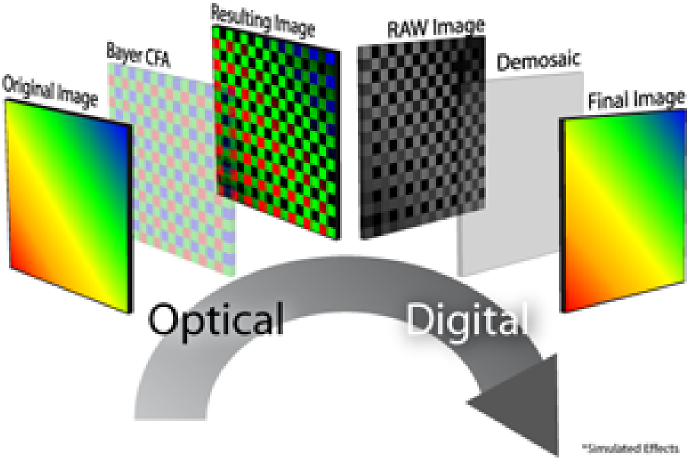
Scalar image u^{CFA} with mosaic effect

$$u^{CFA} = u^R(i,j)m^R(i,j) + u^G(i,j)m^G(i,j) + u^B(i,j)m^B(i,j),$$

$m^R, m^G, m^B \in \{0,1\}$ subsampling functions.

If B denotes the Bayer Filter we have $u^{CFA} = B(u^c)$.

ADMM algorithm for demosaicking deblurring denoising



Acquisition chain

u^c is also degraded by the presence of blur and noise.

We suppose the following form (with abuse of notation) :

$$H = \begin{bmatrix} H & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & H \end{bmatrix}$$

- H is a matrix representing the standard convolution with some Gaussian kernel.
- the noise $b \equiv \mathcal{N}(0, \sigma^2)$ is supposed to be additive and Gaussian.

Acquisition process

$$u^c = [u^R(i,j), u^G(i,j), u^B(i,j)]^T \rightarrow BH(u^c) + b = u^{CFA}$$

Observed image

$$u^{CFA} = BH(u^c) + b$$

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Ill posed Inverse problem

Reconstructing u^c starting from u^{CFA}

A decorrelated basis

RGB components strongly correlated

$\downarrow$

Find a suitable representation $u^d = (\phi, \psi_1, \psi_2)$

$\downarrow$

ϕ, ψ_1, ψ_2 decorrelated

After some manipulations

$$u^c = \psi_0 + (m^R + m^G - m^B)\psi_1 + 2(m^B - m^R)\psi_2$$

$u^L = \psi_0$ (the luminance), $u^{G/M} = \psi_2$ (the green/magenta chrominance) and $u^{R/B} = \psi_1$ (the red/blue chrominance) :

$$u^d = \begin{bmatrix} u^L \\ u^{G/M} \\ u^{R/B} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} u^R \\ u^G \\ u^B \end{bmatrix} = T(u^c)$$

and

$$u^c = \begin{bmatrix} u^R \\ u^G \\ u^B \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} u^L \\ u^{G/M} \\ u^{R/B} \end{bmatrix} = T^{-1}(u^d).$$

Remarks

It is possible to show (Aubert,G.,Blanc-Féraud, Alleyson, Hel-or, Hoyer, Hyvrinen) :

1. Supports of u^L, $u^{G/M}$ $u^{R/B}$ disjoint in the Frequency Fourier domain
2. The coefficients in the new basis are approximately decorrelated ($\rho \simeq 0$)

The Variational method

Observation equation

$$u^c \rightarrow H(u^c) \rightarrow BH(u^c) \rightarrow BH(u^c) + b = u^{CFA}$$

$u^c = T^{-1}(u^d)$

$$u^d \mapsto T^{-1}(u^d) \mapsto HT^{-1}(u^d) \mapsto BHT^{-1}(u^d) \rightarrow BHT^{-1}(u^d) + b = u_0.$$

where we have set $u_0 = u^{CFA}$.

Minimization problem

In the new basis

$$\arg\min_{u^d} \|\nabla u^L\|_1 + \|\nabla u^{G/M}\|_1 + \|\nabla u^{R/B}\|_1 + \mu \|BHT^{-1}(u^d) - u_0\|_2^2.$$

Going back to u^c

Set $u^c = T(u^d)$

ADDM method

constrained minimization problem of the form

$$\min_{u,z} J(z) + G(u) \quad \text{subject to } Ez + Fu = b$$

where $J, G : \mathbb{R}^d \to \mathbb{R}$ and where E and F are matrices.

Augmented Lagrangian

$$L_\alpha(z, u, \lambda) = J(z) + G(u) + \langle \lambda, Fu + Ez - b \rangle + \frac{\alpha}{2} \|Fu + Ez - b\|^2$$

Iterations

$$\begin{cases} (z^{k+1}, u^{k+1}) = \operatorname{argmin}_{z,u} L_\alpha(z, u, \lambda^k) \\ \lambda^{k+1} = \lambda^k + \alpha(Fu^{k+1} + Ez^{k+1} - b), \quad \lambda^0 = 0 \end{cases}$$

Theorem 1 (Eckstein, Bertsekas)

Suppose E has full column rank and $G(u) + \|F(u)\|^2$ is strictly convex. Let λ_0 and u_0 arbitrary and let $\alpha > 0$. Suppose we are also given sequences $\{\mu_k\}$ and $\{\nu_k\}$ with $\sum_k^\infty \mu_k < \infty$ and $\sum_k^\infty \nu_k < \infty$. Assume that

1. $\|z^{k+1} - \operatorname{argmin}_{z\in\mathbb{R}^N} J(z) + \langle\lambda^k, Ez\rangle + \frac{\alpha}{2}\|Fu^k + Ez - b\|^2\| \leq \mu_k$
2. $\|u^{k+1} - \operatorname{argmin}_{z\in\mathbb{R}^M} G(u) + \langle\lambda^k, Fu\rangle + \frac{\alpha}{2}\|Fu + Ez^{k+1} - b\|^2\| \leq \nu_k$

If there exists a saddle point of $L_\alpha(z, u, \lambda)$ then $(z^k, u^k, \lambda^k) \to (z^*, u^*, \lambda^*)$ which is such a saddle points. If no such saddle point exists, then at least one of the sequences $\{u^k\}$ or $\{\lambda_k\}$ is unbounded.

Application to our problem

Our problem

$$\arg\min_{u^d} \|\nabla u^L\|_1 + \|\nabla u^{G/M}\|_1 + \|\nabla u^{R/B}\|_1 + \mu\|BHT^{-1}(u^d) - u_0\|_2^2,$$

ADMM form

$$\min_{u^d, z} J(z) + G(u^d) \quad \text{subject to } Ez + Fu^d = b.$$

$$G(u^d) = 0 \quad z = \begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} \nabla u^d \\ BHT^{-1}(u^d) - u_0 \end{bmatrix},$$

$$E = -I, \quad F = \begin{bmatrix} \nabla \\ \underbrace{BHT^{-1}}_{:=K} \end{bmatrix} \quad \text{where} \quad \nabla = \begin{bmatrix} \nabla^L & 0 & 0 \\ 0 & \nabla^{G/M} & 0 \\ 0 & 0 & \nabla^{R/B} \end{bmatrix},$$

$$b = \begin{bmatrix} 0 \\ u_0 \end{bmatrix} \quad \lambda = \begin{bmatrix} p \\ q \end{bmatrix}$$

$$\begin{aligned} & L_\alpha(z, u^d, \lambda) = \|w\|_1 + \mu\|v\|_2^2 + \langle p, \nabla u^d - w\rangle + \langle q, Ku^d - u_0 - v\rangle \\ + \quad & \frac{\alpha}{2}\|v - Ku^d + u_0\|^2 + \frac{\alpha}{2}\|\nabla u^d - w\|^2. \end{aligned}$$

ADMM Iterations

$$
\begin{aligned}
w^{k+1} &= \underset{w}{\operatorname{argmin}} \|w\|_1 + \frac{\alpha}{2}\|w - \nabla(u^d)^k - \frac{p^k}{\alpha}\|_2^2 \\
v^{k+1} &= \underset{v}{\operatorname{argmin}} \mu\|v\|_1 + \frac{\alpha}{2}\|v - K(u^d)^k + u_0 - \frac{q^k}{\alpha}\|_2^2 \\
(u^d)^{k+1} &= \underset{u^d}{\operatorname{argmin}} \frac{\alpha}{2}\|\nabla u^d - w^{k+1} + \frac{p^k}{\alpha}\|_2^2 \\
&+ \frac{\alpha}{2}\|Ku^d - v^{k+1} - u_0 + \frac{q^k}{\alpha}\|_2^2 \\
p^{k+1} &= p^k + \alpha(\nabla(u^d)^{k+1} - w^{k+1}) \\
q^{k+1} &= q^k + \alpha(K(u^d)^{k+1} - u_0 - v^{k+1}),
\end{aligned}
$$

with $p^0 = q^0 = 0$ and $\alpha > 0$

minimization with respect to w and v

$$w^{k+1} = S_{\frac{1}{\alpha}}(\nabla (u^d)^k + \frac{p^k}{\alpha})$$

$$v^{k+1} = S_{\frac{\mu}{\alpha}}(K(u^d)^k - u_0 + \frac{q^k}{\alpha})$$

$$S_{\frac{1,\mu}{\alpha}}(t) = \begin{cases} t - \frac{1,\mu}{\alpha} sign(t) & |t| > \frac{1,\mu}{\alpha} \\ 0 & \text{otherwise.} \end{cases}$$

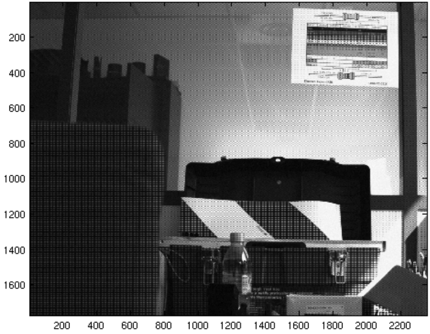
minimization with respect to u^d

$$(u^d)^{k+1} = (-\Delta + K^* K)^{-1} (\nabla^* (w^{k+1} - \frac{p^k}{\alpha}))$$

Numerics

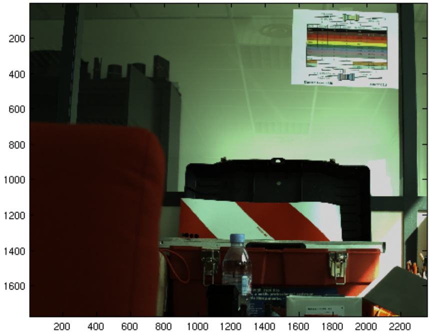
1. pick a color image as a reference u^c, which is a good approximation of a color image without mosaicking effect. ;
2. $$u_0 = BHu^c + b;$$
3. write $u^c = T^{-1}(u^d)$ so that
$$u_0 = BHT^{-1}(u^d) + b;$$
4. Apply the ADMM algorithm to restore u^d ;
5. Set set $u^c = T(u^d)$.

Numerics



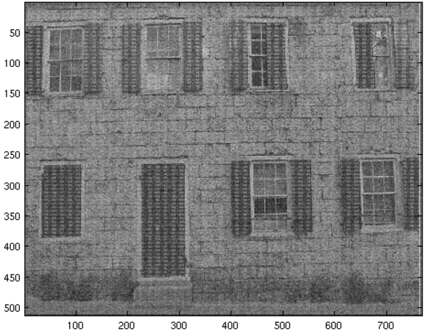
Observed image. Size 2.000 $\times$ 2200 $\sigma = 0.1$.

Numerics



Restored image $u^c = T(u^d)$. $\mu = 10$. CPU time about 30mn

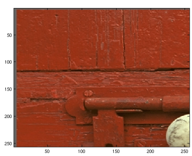
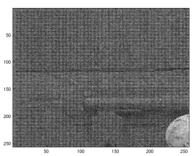
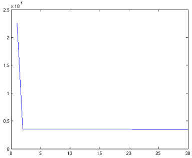
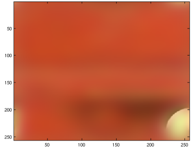
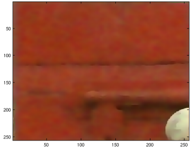
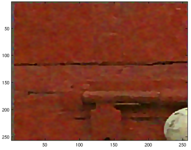
Numerics



Observed image. $\sigma = 0.5$.

Numerics

Restored image $u^c = T(u^d)$. $\mu = 20$.

		
$\mu = 0.5$	$\mu = 5$	$\mu = 50$
		

Role fo the parameter μ. $\sigma = 0.5$.

! ! !Thank you for your attention ! ! !