Kähler and Sasakian Geometry in Rome - KSGR

Titles and abstracts of lectures

Claudio Arezzo, Università di Parma
Complexified Kähler-Ricci flow, K-stability and singularities

Building on previous work by La Nave-Tian, I will present some results concerning the long time behaviour of the Kähler-Ricci flow and the possible singularities it develops, obtained in collaboration with La Nave and Della Vedova-La Nave. In particular I will present some generalizations of Perelman's theorem in the Kähler case when no singularities occur and a discussion of Tian-Donaldson's notion of K-stability.

Olivier Biquard, UPMC-Université Paris 6
Einstein metrics and quaternionic distributions

I shall explain a new construction of Einstein metrics from arbitrary codimension 3 distributions at infinity (joint work with Stuart Armstrong). Special cases were constructed a long time ago by Galicki.

Charles P. Boyer, University of New Mexico
Sasakian Geometry: the recent work of Krzysztof Galicki

I give a review of recent work with our deceased friend and colleague to the memory of whom this conference is dedicated. Particular attention is given to the existence of Sasaki-Einstein metrics on homotopy spheres as well as on simply connected spin 5-manifolds.

Ivan Cheltsov, University of Edinburgh
Kähler-Einstein del Pezzo orbifolds

A sufficient condition for the existence of an orbifold Kähler-Einstein metric on a Fano orbifold (due to Tian, Nadel, Kollar and Demailly) can be formulated in terms of the so-called global log canonical threshold, which is an algebraic counterpart of the alpha-invariant of Tian. For del Pezzo orbifolds this invariant must be strictly greater than 2/3 to ensure the existence of an orbifold Kähler-Einstein metric. Motivated by a problem in Sasakian geometry, Araujo, Boyer, Galicki, Johnson, Kollar and Nakamae studied the problem of existence of an orbifold Kähler-Einstein metric on well-formed quasismooth del Pezzo hypersurfaces in weighted projective spaces. In particular, they classified such surfaces under two additional conditions: the orbifold index is strictly less than 3/2 times the smallest weight of the weighted projective space, the orbifold index is at most 10. The former condition is essential for the sufficient condition of existence of a Kähler-Einstein metric (it also fits well the obstruction for the existence of a Kähler-Einstein metric found by J. Gauntlett, D. Martelli, J. Sparks and S.-T. Yau). We complete the work of Araujo, Boyer, Galicki, Johnson, Kollar and Nakamae. Namely, we show that the assumption that the orbifold index is at most 10 is redundant, and we find all well-formed quasismooth del Pezzo hypersurfaces in weighted projective spaces whose global log canonical threshold is greater than 2/3. As a by-product, we obtain a classification of weakly-exceptional (in a sense of Shokurov) of quasismooth well-formed del Pezzo hypersurfaces in weighted projective spaces.

Xiuxiong Chen, University of Wisconsin-Madison
Space of Ricci flows

We prove 2 dimensional Hamilton-Tian conjecture on Kähler Ricci flow and we use it to prove the Calabi conjecture in Fano-surface (i.e., Tian's celebrated theorem on Calabi conjecture). This is a joint work with my former student Bing Wang.

Liana David, IMAR
Conformal-Killing forms in quaternionic-Kähler geometry

I will begin with a brief survey on conformal-Killing forms, based on the works of A. Moroianu and U. Semmelmann. Then I will present two recent results about conformal-Killing forms in quaternionic-Kähler geometry. The first, proved in collaboration with M. Pontecorvo, provides a characterization of quaternionic projective space in terms of existence of conformal-Killing 2-forms. The second states that an almost quaternionic-Hermitian manifold whose fundamental 4-form is conformal-Killing is quaternionic-Kähler.

José Figueroa-O'Farrill, University of Edinburgh and IPMU, University of Tokyo
M2-branes and Sasakian geometry

We review the rôle played by Sasakian geometry in the AdS4/CFT3 correspondence and motivate a fantasy relating Sasaki-Einstein manifolds to certain kinds of triple systems, based on our recent understanding of three-dimensional superconformal Chern-Simons theories with matter.

Akira Fujiki, Osaka University
On anti-self-dual bihermitian structures on surfaces of class VII

In the recent joint work Pontecorvo and I have found families of anti-self-dual bihermitian structures on many compact complex surfaces of class VII with effective and disconnected anti-canonical divisors, especially on any hyperbolic Inoue surfaces (Inoue-Hirzebruch surfaces) and on certain parabolic Inoue surfaces. In my lecture I would like to talk about various aspects of our results and discuss on some remaining problems and further possible developments.

Akito Futaki, Tokyo Institute of Technology
Hilbert series and obstructions to asymptotic semistability

Given a polarized manifold there are obstructions for asymptotic semistability described as integral invariants. One of them is an obstruction to the existence for the first Chern class of the polarization to admit a constant scalar curvature Kähler (cscK) metric. A natural question is whether or not the other obstructions are linearly dependent on the obstruction to the existence of a cscK metric. The purpose of this talk is to show that this is not the case by exhibiting toric Fano threefolds in which these obstructions span at least two dimension. To see this we show that on toric Fano manifolds these obstructions are obtained as derivatives of the Hilbert series.

Paul Gauduchon, École Polytechnique
Extremal Kähler metrics and stability

It has been generally believed that the existence of Kähler metrics of constant scalar curvature on a compact complex manifold should be equivalent to some kind of stability of the manifold with respect to some polarization. This conjecture can be tested in various particular cases. In this talk, we mainly focus our attention on projective bundles $\mathbb{P}(E) \to S$, where E is a holomorphic vector bundle over a Riemann surface S of genus ≥ 2. We show that $\mathbb{P}(E)$ admits Kähler metrics of constant scalar curvature, in any Kähler class, if and only if the vector bundle E is polystable, i.e. is a sum of stable vector bundles of equal slope, and we provide partial results supporting the conjecture that $\mathbb{P}(E)$ should admit extremal Kähler metrics if and only if E is a sum a stable vector bundles.

Nigel Hitchin, University of Oxford
G2 manifolds and integrable equations

Yau, Zaslow and Loftin studied a family of Calabi-Yau metrics with special Lagrangian torus fibrations generated from solutions of a real form of the Tzitzeica equation. We look at an analogous problem for G2 manifolds with a coassociative fibration. This is work done with my student David Baraglia.

Daniel Jafferis, Rutgers
3-Sasakian duals with flavor

There has been remarkable progress in understanding the infrared dynamics of M2 branes in the past year. This has lead to a correspondence between supersymmetric Chern-Simons-matter theories and M-theory on $\mathrm{AdS}_4 \times 3 - \mathrm{Sasakian}$ 7-manifolds with an extra U(1) isometry. I will discuss some results in the case this U(1) has fixed loci, corresponding to flavor in the dual field theory, as well as some progress in less supersymmetric examples.

Claude LeBrun, Stony Brook University
Einstein Metrics, Complex Surfaces, and Symplectic 4-Manifolds

Which smooth compact n-manifolds admit Einstein metrics? In this talk, I will describe recent progress regarding the 4-dimensional case. These results specifically concern 4-manifolds that also happen to carry either a complex structure or a symplectic structure.

Toshiki Mabuchi, Osaka University
Uniqueness problem for Sasaki-Einstein metrics

In this talk, we shall discuss the recent development about the uniqueness problem for Sasaki-Einstein metrics.

Massimiliano Pontecorvo, Università Roma Tre
On Hermitian geometry of complex surfaces

Recent results with Fujiki are related to generalized Kähler and locally conformally Kähler geometry of complex surfaces.

Yat-Sun Poon, UC Riverside
Generalized contact structures

We explore how one may extend the framework of "generalized complex geometry" from even-dimensional manifolds to odd-dimensional manifolds. Our focus is on integrability and deformation. It is joint work with Aissa Wade of Penn State U.

Martin Rocek, Stony Brook University
Sigma-Models, the c-map, hyperkähler geometry, and a new duality

I briefly review the sigma-model approach to geometry, and then describe the rigid c-map in twistor space, the symplectic duality it gives rise to on semi-flat hyperkähler geometries that arise in the recent work of Gaiotto, Moore, and Neitzke on wall-crossings, and propose a new generalization. I will also review my work with Ulf Lindstrom on twistor space.

Ioana Suvaina, New York University
Einstein metrics and exotic smooth structures on 4-manifolds

The existence or non-existence of Einstein metrics on a topological 4-manifold is strongly related to the differential structure considered. We analyze this dependency for manifolds with small topology. We also consider 4-manifolds with the canonical smooth structure, and for a large class of manifolds we prove non-existence theorems of group invariant metrics. The main techniques used come from Seiberg-Witten theory, the geometry of complex surfaces and symplectic topology.

Craig van Coevering, MIT
Noncompact Calabi-Yau manifolds

We consider some complete Ricci-flat Kähler manifolds. These include asymptotically conical metrics on resolutions of singularities, which are generalizations of the familiar Ricci-flat ALE spaces. Recent developments in Sasaki-Einstein geometry provide abundant examples. Similar metrics on affine varieties are also considered. In particular, we prove any smoothing of a toric Gorenstein singularity has such a metric.

Jeff Viaclovsky, University of Wisconsin-Madison
Limits of constant scalar curvature anti-self-dual metrics

I will discuss some theorems regarding sequences of anti-self-dual metrics limiting to orbifold metrics, and give various examples and connections with the orbifold Yamabe problem.

Guofang Wang, Otto-von-Guericke University Magdeburg
Sasaki-Ricci flow

In this talk I first introduce the Sasaki-Ricci flow and then discuss its convergence on 3-dimensional Sasakian manifolds.

Brian Weber, Stony Brook University
Singularities of Noncollapsed Extremal Kähler Metrics

One would like to understand the degeneration of sequences of canonical metrics, for this can open the way to compactness arguments in the moduli space. In this talk we examine the degeneration of noncollapsed extremal Kähler metrics into singular spaces, and find that singularities are smooth orbifold points that bubble off one-ended ALE instantons. In at least some natural sitations, the "noncollapsing" condition can be verified, and the bubbling can be controlled.