Isabella Birindelli
Curriculum Vitae et Studiorum

Generalities

Born in Dunkerque (France) 31/03/1963, Nationality: italian.
Professional Address: Istituto Matematico “G. Castelnuovo”.
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Education

- **Ph.D. in Mathematics.**
  Courant Institute of Mathematical Sciences, N.Y.U., New York, U.S.A.
  September 1987-September 1992.
  Thesis :*Second order elliptic equations in general domains:Hopf's lemma and Anti maximum principle* .
  Advisor: Louis Nirenberg.
- **Laurea (Master) in Mathematics**, 110/110 Magna cum Laude.
  Università degli studi di Roma “La Sapienza”. January 1987.
  Advisor: Umberto Mosco.

Positions

- November '98-present:
  Associate Professor at the Università di Roma “La Sapienza” member of the Mathematical Department, confirmed in 2001.
- March '92-November '98:
  Researcher at the Università di Roma “La Sapienza”, member of the Mathematical Department
- '90-'92 Teaching Assistant, N.Y.U.

Research field

Elliptic, degenerate elliptic and systems of partial differential equations, in particular I have studied qualitative properties, regularity, existence and non-existence theorems in the semi-linear, quasi-linear and linear cases.

Honors, scholarship and other positions

- Invitation for a month position at the University of Stanford: may 2005.
- May-June 2004: "CNR-Short term mobility" in order to visit F. Demengel
- April 2004 Professeur Invite' at the University of Cergy Pontoise
- Co-organizer with C. Gutierrez and E. Lanconelli of an international conference at Cortona june 2003, subsidized by INDAM: "Non linear sub-elliptic equations"
- 2002/2003 Italian partner of a research project co-funded by CNR and CNRS
- may 2001 and June 2002 Professeur Invite' at the Universite' de Cergy Pontoise
- April 2001: 2 weeks invitation by Rutgers University
- April-July 2000 "Directeur de Recherche" of the CNRS at the Laboratoire of Cergy-Pontoise.
- August 1999, Invited Researcher at the Indian Statistical Institute and at Indian Science Institute, partially funded by C.N.R.
- April 1998, Visiting at CeReMaDe of University of Paris IX, in the TMR Project: "Viscosity solutions and Applications"
- May 1996, Scholarship :**Séjour scientifique de longue durée** given by the Scientific Research Minister of France, for a twelve month stay at the Université de Cergy- Pontoise, Paris, France
- March-April 1996, March 1995 Visiting professor at Ceremade as a Member of the European project "Human Capital and Mobility" **Mathematical modeling of image processing**
- Different scholarship from different institutions to do research abroad or in Italy, such as:

  1990-1992 **Ministero della Pubblica Istruzione**

  1987-90 **Consiglio Nazionale delle Ricerche**

  March 1987- August 1987 **Istituto Nazionale di Alta Matematica "F. Severi"**

Main conferences and seminar, as speaker

- Septemper 2004 “Viscosity, metric and control theoretic methods in nonlinear PDE’s” conference in Serapo
- Giugno 2003 Politecnico di Milano
- Marzo 2003 Laboratoire J.L. Lions, Université Paris VI
- Marzo 2003 Università di Amiens (Organizzatore: A. Farina)
- Novembre 2002 Conference in Campinas, Brasile: ”Nonlinear elliptic equations”
- Giugno 2002 convegno UMI/AMS sezione “Viscosity Solutions”
- Mars 2002 Massachussetts Insititue of Technology
- October 2001 University of Bologna
- September 2001 CIRM Luminy
- April 2001 Rutgers’s University
- May 2000, Université de Cergy Pontoise
- October 1999, AMS Meeting at Providence Rhode Island
- August 1999, Tata Insitute of Fundamental Research, Indian Institute of Science and Indian Statistical Institute, Bangalore, India
- March 1999, Université Paris IX- Dauphine, Parigi,
- February 1999, Universita’ “la Sapienza”, in a one day workshop: “Semi-linear sub-elliptic equations”
- February 1998, Université Paris IX- Dauphine, Parigi,
- January 1998, Workshop “Analisi e Geometria” Université de Cergy-Pontoise
- December 1997, Ecole Polytechnique, Palaiseau, Parigi
- November 1997, Université de Rouen
- May 1997, Workshop “Analisi e Geometria” Université de Cergy-Pontoise
- April 1997, Conference “Analisi non lineare e Calcolo delle Variazioni”, Bologna
- February 1997, “Recenti sviluppi nella teoria delle equazioni alle derivate parziali”, Accademia delle Scienze, Bologna
- January 1997, Colloquium of Mathematics of the Université de Cergy-Pontoise
- May 1996, Conference “Equazioni alle derivate Parziali di tipo Ellittico”, Cortona
- April 1996, École Normale Supérieure workshop “Séminaire d’analyse non linéaire et Applications”
- March 1996, Università di Trieste
- October 1995, Conference “Reaction Diffusion Systems” Università di Trieste
- May 1995, Università di Padova

- May 1994 École Normale Supérieure
- February 1993 Università di Roma 2 "Tor Vergata"

Other research-related activities

Reviewer for the "Mathematical Review", referee for the following journals "Communication in Partial Differential Equations", "Journal of European Mathematical Society", " Proceedings of the Royal Society of Edinburgh", "Pacific Journal","Annali di Pisa", "Non Linear Differential equations and applications", "Nonlinearity" , member of the Gruppo Nazionale di Analisi Funzionale e Applicazioni.

Scientific pubblications

1. *First eigenvalue and Maximum principle for fully nonlinear singular operators.* with F. Demengel, submitted.
2. *Comparison principle and Liouville type results for singular fully nonlinear operators.* con F. Demengel to appear in the Annales de Toulouse.
3. *Sur les équations de Lane-Emden avec opérateurs non linéaires.* (French) [Lane-Emden equations with fully nonlinear operators] with F. Demengel, C. R. Math. Acad. Sci. Paris 336 (2003), no. 9, 725–730.
4. *Homogenization of Hamilton-Jacobi equations in the Heisenberg group* with J. Wigniolle, to appear on Comm. Pure Applied Anal.
5. *A negative answer to a one-dimensional symmetry problem in the Heisenberg group* with E. Lanconelli to appear on Calculus of Variation.
6. *Existence of solutions for semi-linear equations involving the p-Laplacien : the non coercive case* with F. Demengel, Calculus of Variation (2004).
7. *Some Liouville theorems for the p-Laplacian* with F. Demengel, Proceedings of the 2001 Luminy Conference on Quasilinear Elliptic and Parabolic Equations and System, 35–46 (electronic), Electron. J. Differ. Equ. Conf., 8.
8. *On some partial differential equation for non coercive functional and critical Sobolev exponent.* with F. Demengel Differential and Integral Equations 15 (2002), no. 7, 823–837.
9. *A note on one dimensional symmetry in Carnot Groups* with E. Lanconelli, Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl. 13 (2002), no. 1, 17–22.
10. *Superharmonic functions in the Heisenberg group: estimates and Liouville theorems* NoDEA Nonlinear Differential Equations Appl. 10 (2003), no. 2, 171–185.

11. *One dimensional symmetry in the Heisenberg group* with J. Prajapat Annali della Scuola Normale Superiore di Pisa, 2001
12. *A note on one-dimensional symmetry in Carnot groups* with E. Lanconelli, to appear Rendiconti Accademia Nazionale Lincei.
13. *Monotonicity results for Nilpotent stratified groups* with J. Prajapat, Pacific Journal. Pacific J. Math. 204 (2002), no. 1, 1–17
14. *Bifurcation problems for superlinear elliptic indefinite equations* with J. Giacomoni, Topological Methods in Nonlinear Analysis (2000)
15. *Morse Index and Liouville Property for Superlinear Elliptic Equations on the Heisenberg group* with I. Capuzzo Dolcetta, Contributions in honor of the memory of Ennio De Giorgi (Italian). Ricerche Mat. 49 (2000), suppl., 1–15.
16. *Nonlinear Liouville theorems in the Heisenberg group via the moving plane method* with J. Prajapat, Comm. Partial Differential Equations (1999)
17. *Existence and numerical approximation results for a class of quasi-linear elliptic system arising in image segmentation* with S. Finzi Vita, No DEA(1998).
18. *Existence of the principal eigenvalue for cooperative elliptic systems in a general domain*, with E. Mitidieri e G. Sweers, Differential Equations-Differentil'nye Uravnenija (1998)
19. *Liouville theorems for elliptic inequalities and applications*, with E. Mitidieri, Proceedings of the Royal Society of Edinburgh, vol 128A (1998)
20. *Indefinite semi-linear equations on the Heisenberg group: a priori bounds and existence*, with I. Capuzzo Dolcetta e A. Cutrí, Communications in Partial Differential Equations, **23**, (1998).
21. *Nonlinear Liouville theorems*, Proceedings of the Meeting Reaction Diffusion Systems, Lecture Notes in Pure and Applied Mathematics, Marcel Dekker Inc.(1997)
22. *Liouville theorems for semilinear equations on the Heisenberg group*, with I. Capuzzo Dolcetta and A. Cutrí, Annales de l'Institut Henri Poincaré-Analyse non linéaire,**vol 14**, 3 (1997).
23. *Periodic solutions for a class of second order systems with a small forcing term*, Nonlinear Analysis **Vol 27** (1996).
24. *A semi-linear problem for the Heisenberg Laplacian*, with A. Cutrí, Rendiconti del Seminario dell'Università di Padova **Vol.** , (1996).
25. *Hopf's lemma and Anti-maximum Principle in General Domains*, Journal of Differential Equations, **Vol. 119**, (1995).
26. *Non linear two-obstacles problems: Pointwise regularity*, with M.A. Vivaldi, Rendiconti di Matematica, Serie VII **Vol. 14** , (1994).
27. Ph.D. thesis at the Courant Institute: *Second order elliptic equations in general domains:Hopf's lemma and Anti maximum principle* (1992)

28. *Energy decay for Dirichlet problems in irregular domains with quadratic Hamiltonian*, Integral and differential equations, **Vol 8**, (1992).

Description of some of the most recent results ([1], [2], [5],[11],[13],[16])

1) Fully nonlinear operators

In the last two papers ([1] and [2]), I have studied with F. Demengel a large class of fully nonlinear elliptic operators $Lu := F(\nabla u, D^2u)$ which may be singular or degenerate (as the p-Laplacian Δ_p). More precisely they satisfy

(F1) $F(tp, \mu X) = |t|^\alpha \mu F(p, X)$, $\forall t \in \mathbb{R}$, $\mu \in \mathbb{R}^+$, $\alpha > -1$

(F2) $a|p|^\alpha \mathrm{tr}N \leq F(p, M+N) - F(p, M) \leq A|p|^\alpha \mathrm{tr}N$ for $0 < a \leq A$, $\alpha > -1$ and $N \geq 0$.

The class of operators satisfying (F1) and (F2) is large and includes

$$F(\nabla u, D^2u) = |\nabla u|^\alpha \mathcal{M}_{a,A}(D^2u)$$

where $\alpha > -1$ and $\mathcal{M}_{a,A}$ is one of the Pucci operators. Another example is given by

$$F(\nabla u, D^2u) = \Delta_p u$$

with $\alpha = p+1$. Of course the right notion of solution in this context will be that of viscosity solution suitably adapted to this setting (see [2]).

After obtaining a comparison principle for sub and super viscosity solution for a class of operators satisfying (F2), we have introduced a notion of *first eigenvalue* for fully nonlinear operators which are non variational but homogeneous (F1). Following Berestycki, Nirenberg and Varadhan this so called eigenvalue will be defined through the *Maximum Principle.*

Before going into details, let us mention that lately a certain number of interesting papers have appeared which treat viscosity solutions for equations involving the p-Laplacian. In fact Juutinen, Lindqvist and Manfredi opened the way to this topic. We would like to emphasize that in those papers the point of view is really on the p-Laplacian and its variational structure is used. This is not the case here since the operators we consider are fully nonlinear.

The first key ingredient is the following:

Theorem 0.1 *Suppose that Ω is a bounded open piecewise $\mathcal{C}^1$ domain of $\mathbb{R}^N$. Suppose that for $\lambda \in \mathbb{R}$ there exists a function $v > 0$ such that $F(\nabla v, D^2 v) + \lambda v^{\alpha+1} \leq 0$ in Ω. Then, for $\tau < \lambda$, every viscosity solution of*

$$\begin{cases} F(\nabla\sigma, D^2\sigma) + \tau|\sigma|^\alpha\sigma \geq 0 \text{ in } \Omega \\ \sigma \leq 0 \text{ on } \partial\Omega \end{cases}$$

satisfies $\sigma \leq 0$ in Ω.

This theorem allows us to define

$$\bar{\lambda} = \sup\{\lambda \in \mathbb{R},\ \ \exists\, \phi > 0 \ \text{ in } \Omega,\ F(\nabla\phi, D^2\phi) + \lambda\phi^{\alpha+1} \leq 0 \text{ in the viscosity sense }\}.$$

In other words, if we adopt the notation $I_\alpha(u) = |u|^\alpha u$, then we can say that $\bar{\lambda}$ is the supremum of the value λ such that $F + \lambda I_\alpha$ satisfies the Maximum Principle in Ω.

The value $\bar{\lambda}$ has the following features that justify the name of eigenvalue:

Theorem 0.2 *There exists ϕ a continuous positive viscosity solution of*

$$\begin{cases} F(\nabla\phi, D^2\phi) + \bar{\lambda}\phi^{\alpha+1} = 0 & \text{in } \Omega \\ \phi = 0 & \text{on } \partial\Omega. \end{cases}$$

Furthermore

Theorem 0.3 *For $\lambda < \bar{\lambda}$ if $f < 0$ in Ω and bounded then there exists a unique u nonnegative viscosity solution of*

$$\begin{cases} F(\nabla u, D^2 u) + \lambda u^{\alpha+1} = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{0.1}$$

Let us notice that in order to prove the Theorems 0.2 and 0.3 we need to obtain some estimates which are interesting in their own right:

Theorem 0.4 *Suppose that f is a bounded function in $\bar{\Omega}$ then if u is a bounded non-negative viscosity solution of $F(\nabla u, D^2 u) = f$ in Ω, it is Hölder continuous:*

$$|u(x) - u(y)| \leq M|x - y|^\gamma.$$

The proof is inspired by a paper of Ishii and Lions.

2) Qualitative properties of semi-linear equations in the Heisenberg group. Gidas, Ni, Nirenberg in their celebrated work have used the moving planes method to prove the symmetry of positive solutions of Dirichlet problem in bounded domains for

$\Delta u + f(u) = 0$ if f is Lipschitz continuous. The main features of the Laplacian used in the proof are the maximum principle and the invariance with respect to reflexion.

On the other hand, monotony properties of the solutions have been proved by Berestycki and Nirenberg, through the so called sliding method that uses the invariance with respect to translation.

In collaboration first with Prajapat then with Lanconelli we have considered the question of symmetry of solutions for semi linear equations in the Heisenberg group. In particular, using the invariance with respect to the group action in [11] we have proved the so called "Gibbons conjecture"

Theorem: *Let u be a bounded solution of*

$\Delta_H u + u - u^3 = 0$ *in* H^N.

If x_1 is a horizontal direction and $\lim_{x_1 \to \pm\infty} u = \pm 1$ *uniformly with respect to the other variables then* $u(\xi) = U(x_1)$.

The proof uses the sliding method and a Maximum Principle in unbounded domains. This Maximum Principle is interesting by itself and requires a delicate construction of some barrier functions.

It is natural to wonder what happens if the limit condition is not in the horizontal direction but in the anisotropic direction (which we will denote by t.) Let us mention that if $u = U(t)$ then u is not a solution of the equation. Hence we cannot obtain an equivalent theorem in that direction. On the other hand it is possible to prove that the solutions are monotone. Hence generalizing to the Heisenberg setting, a conjecture by De Giorgi a natural question becomes:

Are the level sets of bounded monotone solutions of $\Delta_H u + u - u^3 = 0$ *in* H^N *hyper-planes?*

De Giorgi's conjecture is also called ε-version of Bernstein Theorem. From this point of view the right question would be:

Are the level sets of bounded monotone solutions of $\Delta_H u + u - u^3 = 0$ *in* H^N *minimal surfaces?*

This question is natural since in [5], we prove that monotone solutions of $\Delta_H u + u - u^3 = 0$ in H^N are local minima of the energy and the blow-in of these solutions converges in L^1 to the characteristic function of a set that has minimal perimeter.

However and this is one of the main result of [5] the answer to both questions is **negative**. Indeed we construct a solution which is monotone in t, which is cylindrically symmetric and whose level sets are not hyper-planes. Furthermore we prove that the only minimal surfaces that are cylindrically symmetric are hyperplanes.

We study other symmetry results in [13] and [16]. It is well known that the Heisenberg Laplacian is a degenerate elliptic second order operator which is invariant with respect to the intrinsic group action and it satisfies the Maximum Principle. We shall also recall that if u is "radial" (in the sense of Korany balls) than $\Delta_H u$ is not radial; while if u is radial in the "horizontal" variables then $\Delta_H u$ has the same symmetry.

Hence it is reasonable to ask the following question:

If Ω is a bounded domain cylindrically symmetric and f is Lipschitz continuous is it true that the positive solutions of a $\begin{cases} \Delta_H u + f(u) = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$ *are cylindrically symmetric?*

To my knowledge, in this generality this is still an open problem but in [13] and [16] we have proved:

Theorem *Let u be a cylindrically symmetric positive solution of* $\Delta_H u + f(u) = 0$

1. *In a cylindrical domain $\Omega = \{(z,t); \psi(|z|,t) < 0\}$ with ψ even in t then u is even in t if f is Lipschitz.*
2. *if $f(u) = u^p$ in H^n then $u \equiv 0$ per $p < \frac{2n+4}{2n}$*

Remark: 1) For $p = \frac{2n+4}{2n}$ Jerison and Lee have proved that all solutions of $\Delta_H u + u^p = 0$ in H^n are cylindrically symmetric and they are all similar up to the group action.

2) For $p \leq \frac{2n+2}{2n}$ there are no non negative solutions besides the trivial one (see my work with Capuzzo Dolcetta and Cutri [22])

3) Garofalo and Vassileev have extended our results to Carnot groups of step 2.