26/10/23

I) Let $f(x) = \begin{cases} x^2 - 3x & x \neq \frac{1}{n} \\ \frac{1}{n^2} & x = \frac{1}{n} \end{cases}$

a) Determine if the limit of f to zero: $\lim_{x\to 0} f(x)$ exists

b) Determine if f is continuous in 0

II) Using the definition of limit prove that

$$\lim_{x\to 0} |x+1| = 2 \quad \text{and} \quad \lim_{x\to +\infty} |x+1| = +\infty$$

III) Using the special limits and the algebra of limit, compute the folloing limits:

a) $\lim_{x\to +\infty} \frac{x+e^x}{x^3+e^{3x}}$ b) $\lim_{x\to +\infty} \frac{\sqrt{x^2+x}-x}{\sqrt{x}}$

c) $\lim_{x\to 0} \frac{x^2+e^{-\frac{1}{x^2}}}{x^3+2^{-\frac{1}{x^2}}}$ d) $\lim_{x\to 0^+} x \log\left(\frac{1}{x}\right)$ e) $\lim_{x\to 0} x^3 e^{\frac{1}{x^2}}$

IV Let $f(x) = \begin{cases} x^2 + \sin^2 x & x > 0 \\ ax + b & x \leq 0 \end{cases}$ for some $a \in \mathbb{R}$ and $b \in \mathbb{R}$

Determine for which $a \in \mathbb{R}$ and $b \in \mathbb{R}$ the folloing holds

a) f is continuous in $\mathbb{R}$

b) $\lim_{x\to -\infty} f(x) = 0$

c) $\lim_{x\to 0} f(x)$ doesn't exist

d) f is bounded from below

V) Let $f(x) = \min\left((1-x^2)^2, 1-x^2\right)$. Determine if f is continuous. Compute $f(0)$ and $f\left(\frac{1}{2}\right)$.