Calculus-Unit 1

Applied Computer Science for AI

Blank examination

Voto finale

Postazione:

Cognome:

Nome:

Matricola:

Canale:

Esercizio	Punteggio
1	
2	
3	
4	
Risp. Mult.	
Totale	

Es. 1 [1+2+1 Points] Given the sequence $a_n = \frac{x^2}{3n^2-2}$ for $n \in \mathbb{N}^*$
a) Compute a_1 and a_2
b) Prove that the sequence is bounded
c) Prove that the sequence is monotone decreasing

Es 2 [3 Points] Given $a \in \mathbb{R}$ and $b \in \mathbb{R}$, let $f(x) = \begin{cases} \frac{\log(1+2x)}{3x} & \text{for } x > 0 \\ a(x+1)^2 + b & \text{for } x \leq 0 \end{cases}$ Determine a and b such that f is differentiable in $\mathbb{R}$.

Es 3 [4 points] Compute the following limit (justify your answer) $\lim_{x\to 1} \dfrac{e^{x^2-1}-1}{\tan(\frac{\pi}{4}x^3)\log(x)}$

Es 4 [1+2+1+2+1 points] Given the function $f(x) = \arctan\left(\frac{x-2}{2x+4}\right)$. Determine:
a) Domain:
b) The limits at the boundary of the domains
c) The asymptotes
d) The derivative
e) The intervals of monotonicity

Es 5 [2 o -1 points] The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = |\log(x)|$

(A) Has a minimum and a maximum (**B**) Has a maximum but no minimum

(C) Has a minimum but no maximum (D) Its minimum is at infinity

Es 6 [2 o -1 punti] The derivative of $f(x) = \sin(2x)e^{\cos(2x)}$ is:

(A) $-4\cos(2x)\sin(2x)e^{\cos(2x)}$ (B) $2e^{\cos(2x)}(\cos(2x) - \sin^2(2x))$ (C) $e^{\cos(2x)}(2\cos(2x) + 4\sin^2(2x))$

(D) $e^{\cos(2x)}(2\cos(2x) + \sin(2x))$ (E) None of the previous answers is correct

Es 7 Let $f : [0, 1] \to \mathbb{R}$ a continuous function. Then

(A)[1/2] The image of f is a closed and bounded interval **T** **F**
(B)[1/2] If $f(0) = f(1)$ then either the maximum of the minimum are reached in the open interval $(0, 1)$ **T** **F**
(C)[1/2] The function reaches all the values between $f(0)$ and $f(1)$. **T** **F**
(D)[1/2] The function reaches only the values between $f(0)$ and $f(1)$. **T** **F**
(E)[1/2] If f is convex in $[0, 1]$, then the graph is below the strait line given by the equation $y = (f(1) - f(0))(x - 1) + f(1)$ **T** **F**

Es 8 Given the equation $z^6 = 1 + i$ in $\mathbb{C}$

(A) It has 2 solutions in $\mathbb{C}$ **T** **F**
(B) The solutions are on the circle of center 0 and radius $\sqrt{2}$ **T** **F**
(C) There exists a solution in $\mathbb{R}$ **T** **F**
(D) The solutions are at the vertices of an hexagon **T** **F**

Es 9 [3 o -1 punti] Let a_n be a bounded sequence. Then necessarily

(A) The sequence has a limit **T** **F**. (B) The sequence is monotone **T** **F**

(C) There exists a converging subsequence **T** **F** (D) All subsequences converge **T** **F**

Es 10 Let $z = \frac{1}{2+3i}$.

(A)[1/2] Then $z_o = \frac{1}{13}(2 - 3i)$ **T** **F**

(B)[1/2] Then $z_o \cdot \overline{z}_o = 13$ **T** **F**

(C)[1/2] $(1 + i)z_o = \frac{1}{13}(5 - i)$ **T** **F**

(D)[1/2] $(z_o)^{-1} = 2i + 3$ **T** **F**