

**Exercise 1: (Taylor's polinome )**  
**Prove that the following equalities hold**

$$\textcircled{1} \quad \sin^2(x) = x^2 - \frac{x^4}{3} + o(x^5) \quad x \rightarrow 0$$

$$\textcircled{2} \quad \cos^2(x) = 1 - x^2 + \frac{x^4}{3} + o(x^5) \quad x \rightarrow 0$$

$$\textcircled{3} \quad \frac{1}{1+e^x} = \frac{1}{2} - \frac{x}{4} + o(x^2) \quad x \rightarrow 0$$

$$\textcircled{4} \quad \log(\cos x) = -\frac{1}{2}x^2 - \frac{1}{12}x^4 + o(x^4) \quad x \rightarrow 0$$

$$\textcircled{5} \quad e^{\sin(x)} = 1 + x + \frac{x^2}{2} - \frac{1}{8}x^4 + o(x^4) \quad x \rightarrow 0$$

**Exercise 2: Study and draw the graph of the following functions:**  
**(domain of definition, value of the limits at infinity, asymptotes, derivative, interval of monotonicity, second derivative interval of convexities)**

$$\textcircled{1} \quad f(x) = (x-1)^3(2-x)$$

$$\textcircled{2} \quad f(x) = \frac{x^2}{\log|x| - 1}$$

No second derivatives

$$\textcircled{3} \quad f(x) = \log(x) - \operatorname{arctg}(x-1)$$

No second derivatives

$$\textcircled{4} \quad f(x) = e^{\frac{x-2}{x}}$$

$$\textcircled{5} \quad f(x) = e^{-|x|} \sqrt{x^2 - 5x + 6}$$

**No second derivatives**