SEQUENCES

6/10/23

I) Determine if the following sequences are monotone:

I.a) $a_n = \frac{1}{n+3}$

I.b) $a_n = \sin\left(\frac{\pi}{n}\right)$.

I.c) $a_n = n^2 - n$

I.d) $a_n = \cos\left(n\frac{\pi}{12}\right)$

I.e)* $a_n = \left(1+\frac{1}{n}\right)^n$

suggestion: compute $\frac{a_{n+1}}{a_n}$.

II) Prove, using the definition of convergence or divergence, the following limits

II.a) $\lim_{n\to+\infty} \sqrt{n} = +\infty$

II.b) $\lim_{n\to+\infty} \frac{2n+1}{n+3} = 2$.

II.c) $\lim_{n\to+\infty} \sqrt{n^2+2} - n = 0$

III) Prove that $a_n = (-1)^n \cdot n$ is neither convergent nor divergent.

IV) Let $a_n = \frac{4n^2+2}{3n^2+1}$. Prove that $\{a_n\}$ is bounded.

(V) Let $a_n = \frac{n+2}{n+1}$. Let $b_n = a_{n+1} - a_n$.

Prove that $\lim_{n\to+\infty} b_n = 0$

VI. In general, let $\{a_n\}$ such that $\lim_{n\to+\infty} a_n = a \in \mathbb{R}$

Prove that $\lim_{n\to+\infty} b_n = 0$ for $b_n = a_{n+1} - a_n$.