

1) Determine the max and minimum of the following sets

$$1.1) E = \{x \in \mathbb{R} / x = (-1)^n + 4, n \in \mathbb{N}\}$$

$$1.2) E = \{x \in \mathbb{R} / x^2 - 3x \leq 2\}$$

2) Prove that:

$$a) 1 = \sup \left\{ 1 - \frac{1}{n} ; n \in \mathbb{N} \setminus \{0\} \right\}$$

$$b) 0 = \inf \left\{ \frac{1}{3n^2} ; n \in \mathbb{N} \setminus \{0\} \right\}$$

$$c) 0 = \sup \left\{ \frac{1}{n} - \frac{1}{m}, n \in \mathbb{N} \setminus \{0\}, m \in \mathbb{N} \setminus \{0\} \right\}$$

$$-1 = \inf \left\{ \frac{1}{n} - \frac{1}{m}, n \in \mathbb{N} \setminus \{0\}, m \in \mathbb{N} \setminus \{0\} \right\}$$

$$(\mathbb{N}^* = \mathbb{N} \setminus \{0\})$$

3) Let $A = \left\{ \frac{1}{n+2}, n \in \mathbb{N} \right\}$, $B = \left\{ \frac{1}{3n+1}, n \in \mathbb{N} \right\}$

• Determine $A \cap B$.

• Does $\frac{1}{5} \in A \cup B$?

• Determine $A \cap \left[-\frac{1}{2}, \frac{1}{5} \right]$

4) Prove that the cardinality of $A =$ cardinality of \mathbb{N} .

5) Let $\mathbb{C} = \{(a, b), a \in \mathbb{R}, b \in \mathbb{R}, (a, b) + (c, d) = (a+c, b+d)\}$
and that \mathbb{C} is a field with $(0, 0)$ and $(1, 0)$ as neutral numbers
and $(a, b) \cdot (c, d) = (ac - bd, ad + bc)$