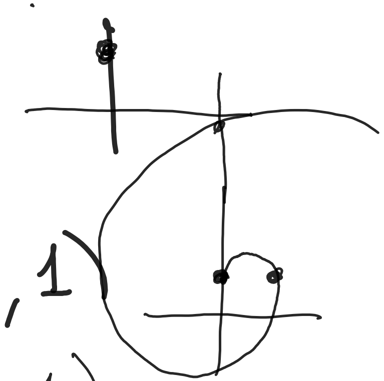
mercoledì 25 novembre 2020 15:42

Esercizio 1



a) $\varphi(t) = (t\cos 4t,\ 1 + t\sin 4t)$

$\varphi(0) = (0\cos 4\cdot 0,\ 1 + 0\cdot\sin 4\cdot 0) = (0, 1)$

$\varphi\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\cos 2\pi,\ 1 + \frac{\pi}{2}\sin 2\pi\right) = \left(\frac{\pi}{2}, 1\right)$

Per determinare se φ è chiuso: $\varphi(2\pi) \overset{?}{=} \varphi(0)$

$\varphi(2\pi) = (2\pi\cos 8\pi,\ 1 + 2\pi\sin 8\pi) = (2\pi, 1) \neq (0, 1)$

la curva <u>non</u> è chiusa

b) $(0,2) \in \varphi \iff$ esiste t t.c.

$$\begin{cases} 0 = t\cos 4t \\ 2 = 1 + t\sin 4t \end{cases}$$

(*) $\begin{cases} 0 = t\cos 4t \\ 1 = t\sin 4t \end{cases}$ | OSS La prima eq. si annulla per $t = 0$ o per t.c. $\boxed{\cos 4t = 0} \leftarrow$

$\sin 4t = -1$

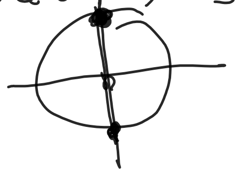
$\frac{\pi}{8}, \frac{5\pi}{8}, \dots$ $\Longrightarrow \sin 4t = 1$ ...

$4t = \frac{\pi}{2}$ $4t = \frac{5\pi}{2}$

$4t = \frac{3\pi}{2}$ ...

$\longrightarrow 1 = t$ ma $\cos 4 \cdot 1 = \cos 4 \neq 0$

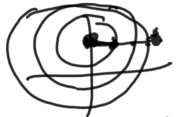
$1 = -t$ IMP perché $t \in [0, 2\pi]$



Non esiste una soluzione di (*).

$t = 1 \longrightarrow \cos 4t = \cos 4 \neq 0$

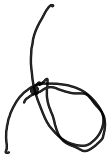
$(0,2) \notin$ alla curva.



d) $\varphi(t) - (0,1) = (t\cos 4t,\ t\sin 4t)$.

$|\varphi(t) - (0,1)| = \sqrt{t^2\cos^2 4t + t^2\sin^2 4t} = \sqrt{t^2(\cos^2 4t + \sin^2 4t)} = \sqrt{t^2} = t$

La distanza tra la curva e il punto $(0,1)$ è $\boxed{t}$, quindi è strettamente monotona crescente, quindi non ci possono essere due $t_1 \neq t_2$ tali che $\varphi(t_1) = \varphi(t_2)$ quindi la curva è semplice.



$\varphi(t_1) = \varphi(t_2)$

$\Downarrow$

$|\varphi(t_1) - (0,1)| = |\varphi(t_2) - (0,1)|$

$t_1 \neq t_2 \Rightarrow t_1 < t_2 \longrightarrow |\varphi(t_1) - (0,1)| < |\varphi(t_2) - (0,1)|$

$$\begin{cases} t_1 \cos 4t_1 = t_2 \cos 4t_2 \\ \cancel{1+} t_1 \sin 4t_1 = \cancel{1+} t_2 \sin 4t_2 \end{cases} \overset{?}{\Longrightarrow} \boxed{t_1 = t_2}$$

$$\tan 4t_1 = \tan 4t_2 \Longrightarrow \begin{cases} t_1 = t_2 \\ 4t_1 = 4t_2 + \pi \quad \text{NO} \\ t_1 = t_2 + \frac{\pi}{4} \end{cases}$$

$t_2 < t_1 \rightarrow \ldots$

$(t_2 + \frac{\pi}{4}) \cos(4t_2 + \pi) = (t_2 + \frac{\pi}{4}) \cos(4(t_2 + \frac{\pi}{4})) \overset{?}{=} t_2 \cos 4t_2$ $\quad 2t_2 = -\frac{\pi}{4} < 0$

$\quad\quad \overset{\shortparallel}{-}(t_2 + \frac{\pi}{4}) \cancel{\cos 4t_2} \overset{?}{=} t_2 \cancel{\cos 4t_2} \quad t_2 = -(t_2 + \frac{\pi}{4})$ $\quad \underline{\underline{IMP.}}$

$4t_1 = 4t_2 + 2\pi$

$t_1 = t_2 + \frac{\pi}{2} \longrightarrow (t_2 + \frac{\pi}{2}) \cos(\cancel{4t_2 + 2\pi}) = t_2 \cancel{\cos(4t_2)}$ IMP

IMP.

$4t_1 = 4t_2 + 3\pi \ldots$

$\boxed{4t_1 = 4t_2 + k\pi}$ $\quad t_1 = t_2 + \frac{k\pi}{4}$ $\quad \varphi(t_2) = \varphi(t_1)$, $t_2 \neq t_1$

$$(t_2 + \frac{k\pi}{4}) \cos(4t_2 + k\pi) = t_2 \cos(4t_2)$$

k è pari $\longrightarrow$ $t_2 + \frac{k\pi}{4} = t_2$ IMP.

k è dispari $\quad -(t_2 + \frac{k\pi}{4}) = t_2 \rightarrow t_2 = -\frac{k\pi}{8}$ IMP.

Una possibile strategia è di calcolare

$$\varphi'(t) = (x'(t), y'(t))$$

$x'(t) > 0$ oppure

se per caso ...

$y'(t) > 0$ allora φ è semplice

$$\varphi(t) = \left(t^2+1,\ t^3 + \log(\cos t)\right)$$

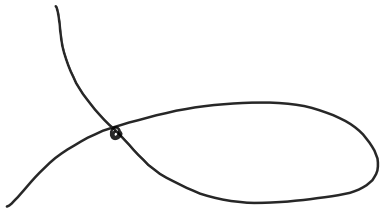
$t \in [0, 4\pi]$

$$\varphi'(t) = \left(\underline{2t},\ 3t^2 - \frac{\sin t}{\cos t}\right)$$

$t > 0 \implies 2t > 0$

$x(t)$ monotona crescente

φ è semplice



$$\begin{cases} x(t_1) = x(t_2) \\ y(t_1) = y(t_2) \end{cases} \overset{?}{\implies} t_1 = t_2$$

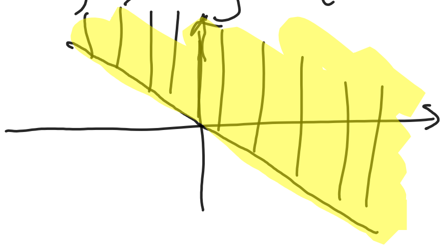
Esercizio 2:

a) $f(x,y) = \sqrt{x+y}$, $f(1,3)$ e dominio

$f(1,3) = \sqrt{1+3} = \sqrt{4} = 2$

$f(1,5) = \dots$

$D = \{(x,y),\ x+y \geq 0\} = \{(x,y),\ y \geq -x\}$



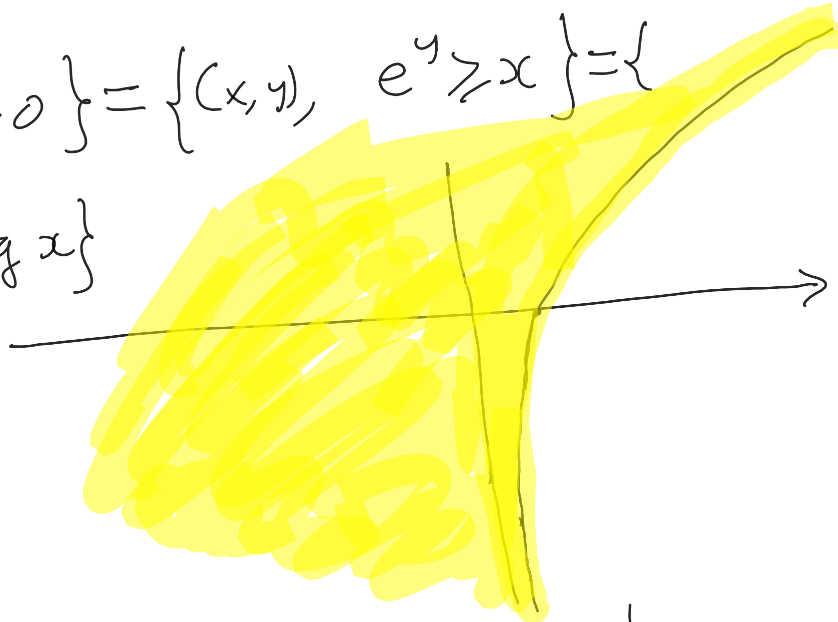
dominio

b) $f(x,y) = \sqrt{e^y - x}$, $f\left(\frac{1}{2}, 0\right)$ – dominio

$f\left(\frac{1}{2}, 0\right) = \sqrt{e^0 - \frac{1}{2}} = \sqrt{1 - \frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$D = \{(x,y),\ e^y - x \geq 0\} = \{(x,y),\ e^y \geq x\} = \{$

$= \{(x,y),\ y \geq \log x\}$



dominio

bordo incluso

$\log\left(x^2 - y^2\right)$ $f\left(1, \frac{1}{5}\right)$ e dominio

c) $f(x,y) = \ldots \left(\frac{\ldots}{y-(1-x^2)^2} \right)$

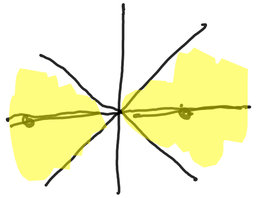
$$f\left(1,\tfrac{1}{2}\right) = \log\left(\frac{1-\frac{1}{4}}{\frac{1}{2}-(1-1^2)^2} \right) = \log\left(\frac{\frac{3}{4}}{\frac{1}{2}} \right) = \log\left(\frac{3}{2} \right)$$

$$D = \left\{ (x,y),\ \frac{x^2-y^2}{y-(1-x^2)^2} > 0,\ y-(1-x^2)^2 \neq 0 \right\}$$

Studiare il segno di x^2-y^2 e il segno $y-(1-x^2)^2$

Cerchiamo dove si annullano.

$x^2-y^2=0 \longrightarrow x^2=y^2$ $\to \boxed{x=y}$, $\to x=-y$

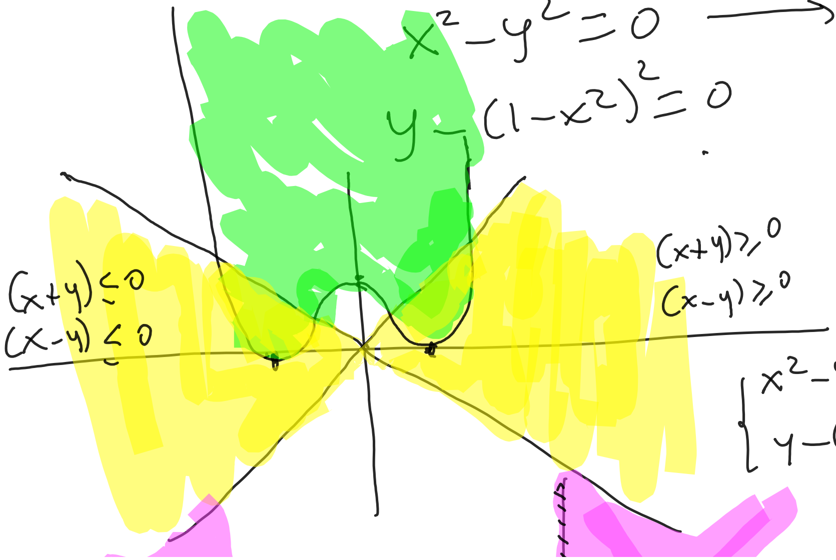


$x^2-y^2=(x$

$y-(1-x^2)^2=0$

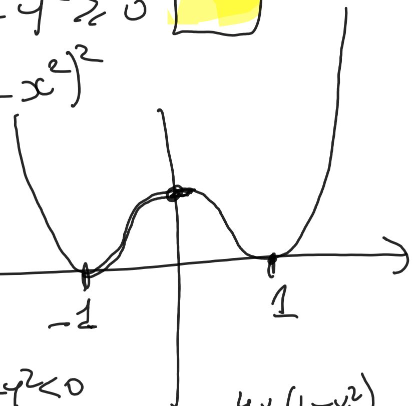
$x^2-y^2 \geq 0$

$y=(1-x^2)^2$

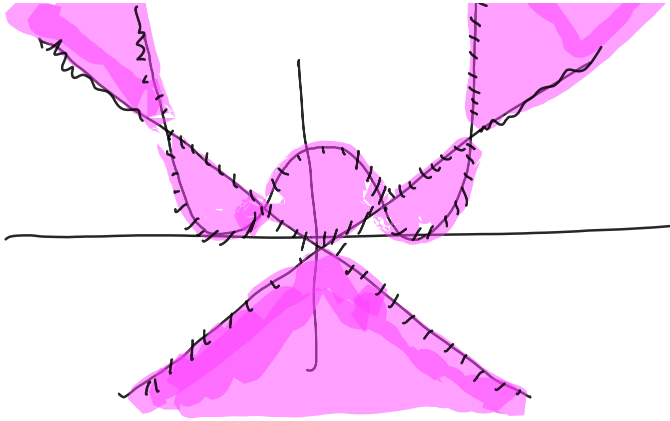


$$\begin{cases} x^2-y^2>0 \\ y-(1-x^2)>0 \end{cases}$$

$$\begin{cases} x^2-y^2<0 \\ y-(1-x^2)^2<0 \end{cases}$$



$4x(1-x^2)$



dominio

senza i bordi

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d) $f(x,y) = \cos\sqrt{x^2+y^2}$, $f(1,3)$ e livello 0 e -1

$f(1,3) = \cos\sqrt{1+9} = \cos\sqrt{10}$

livello: (x,y) t.c. $\cos\sqrt{x^2+y^2} = 0$

$\sqrt{x^2+y^2} = \frac{\pi}{2} + k\pi$ cerchio di centro $(0,0)$ e raggio $\frac{\pi}{2} + k\pi$, $k \in \mathbb{N}$