Lezione del 28/10/2020

Es I — Dom di esistenza

a) $f(x,y)=\dfrac{1}{x+y}$ ✗

$\boxed{\text{b) } f(x,y)=\log[(x^2-1)(x+y)]}$

c) $f(x,y)=\dfrac{\sqrt{y^2-x}}{3x+2y}$

d) $f(x,y)=\sqrt{x^2+y^2-16}+\dfrac{1}{\log(xy)}$

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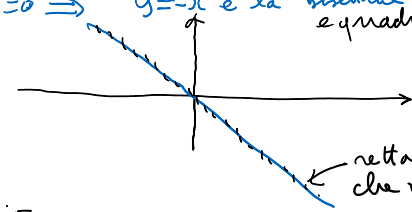
a) $f(x,y)=\dfrac{1}{x+y}$ 1°) Condizione di esistenza

Trovare tutti i valori (x,y) t.c. riesco a calcolare la funzione.

$$\boxed{\begin{array}{l} \frac{1}{x} \\ \log x \\ \sqrt{x} \\ \operatorname{tg} x \end{array}} \begin{array}{l} \to x\neq 0 \\ \to x>0 \\ \to x\geq 0 \\ \to x\neq \frac{\pi}{2}+k\pi \quad \forall k\in\mathbb{N} \end{array}$$

Richiesta: $\boxed{x+y\neq 0}$ scrivetela

Se (x,y) sono dei punti del piano
$x+y=0 \Longrightarrow y=-x$ è la bisettrice del secondo e quadrante



Dominio
$\mathbb{R}^2$ meno la retta.

• $f(x,y)=\dfrac{1}{x^3-y^4}$

Condizione $x^3-y^4\neq 0$

Quali sono i punti che verificano la condizione? cerchiamo i punti che verificano

$\boxed{x^3-y^4=0}$

$y^4=x^3 \to$ 1) $\boxed{y=x^{3/4}}$

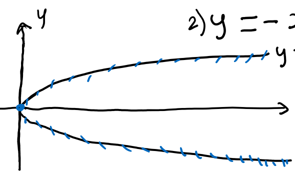
2) $y=-x^{3/4}$

$g(x)=x^{3/4}$
ha come grafico
$y=x^{3/4}$

$D_g=[0,+\infty)$

$g(0)=0$

$g'(x)=\frac{3}{4}x^{-\frac{1}{4}}=\dfrac{3}{4\sqrt[4]{x}}>0$



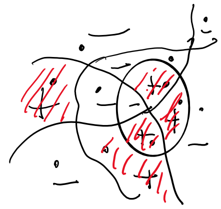
x^α

$\frac{1}{2}<\frac{3}{4}<1$ $\boxed{y=g(x)}$

b) $\boxed{f(x,y) = \log\left((x^2-1)(x+y)\right)}$ non è definita in $(x^2-1)(x+y)=0$;

Condizione $\boxed{(x^2-1)(x+y) > 0}$ (*)

Per quali (x,y) è verificata (*) ?



$(x^2-1)\cdot(x+y)=0$

Il prodotto è nullo se uno dei termini è nullo

$x^2-1=0 \longrightarrow x^2=1$ → $\boxed{x=1}$ $\boxed{x=-1}$

$x+y=0$

$\downarrow$

$y=-x$

In blu sono i pti t.c. $(x^2-1)(x+y)=0$

La funzione $(x^2-1)(x+y)$ ha segno costante in ogni zona!

<0 → NON APPARTIENE

Sostituisco $(0,1)$ in $(x^2-1)(x+y)$ cioè $x=0$, $y=1 \longrightarrow (0-1)(0+1) = -1\cdot 1 = -1 < 0$

→ APPARTIENE

Sostituisco $(2,0)$ in $(x^2-1)(x+y)$

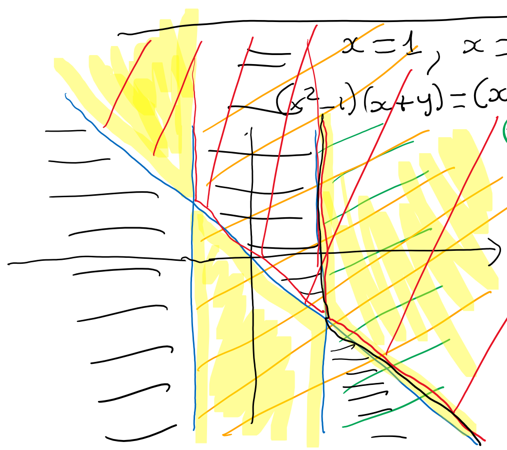
$x=2$, $y=0 \longrightarrow (2^2-1)(2+0) = 3\cdot 2 = 6 > 0$

Sostituisco $(2,-3)$ in $(x^2-1)(x+y)$

$x=2$, $y=-3 \rightarrow (2^2-1)(2-3) = 3\cdot(-1) = -3 < 0$

$x=1$, $x=-1$, ~~rette~~ $y=-x$

$(x^2-1)(x+y) = (x-1)(x+1)(x+y)$



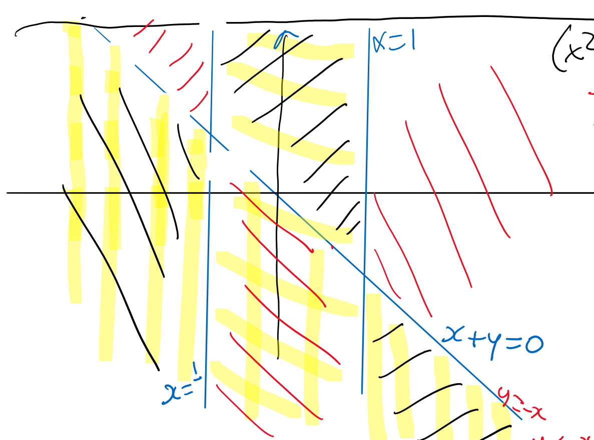
+	+	+	) +	OK
+	−	−		
−	−	+		OK
−	+	−		OK
−	+	+	→	= NO
+	−	+		
+	−	−		= NO
−	+	−	→	=

dominio

$x-1>0$ $x>1$

$x+1>0$ $x>-1$

$y+x>0$ $y>-x$



$(x^2-1)(x+y)$

$x^2>1$

$x>1$ e $x<-1$

$\boxed{x^2-1} < 0$

$\boxed{x+y} < 0$

$y<-x$

+	+	→	+
−	+	→	−
+	+	→	+
−	+	→	−
−	−	→	+
+	−	→	−

NON APPARTIENE

APPARTIENE

a) $\lim\limits_{(x,y)\to(0,0)} xy^2+x-y = 0$

la funzione $f(x,y)=xy^2+x-y$ è un polinomio pertanto è continua in $\mathbb{R}^2$ e $f(0,0)=0$

b) $\lim\limits_{(x,y)\to(1,-1)} \dfrac{x^2-y^2}{2x+2y}$

c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^2+y^2}$

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b) $\lim\limits_{(x,y)\to(1,-1)} \boxed{\dfrac{x^2-y^2}{2x+2y}} = \lim\limits_{(x,y)\to(1,-1)} \boxed{\dfrac{x-y}{2}} = \dfrac{1-(-1)}{2} = 1$

$2x+2y \longleftrightarrow \begin{matrix} x=1 \\ e\ y=-1 \end{matrix} \longrightarrow 2\cdot 1+2(-1) = 2-2=0$

$(1,-1) \notin \mathrm{Dom}\left\{\dfrac{x^2-y^2}{2x+2y}\right\}$

$x^2-y^2 \longrightarrow \begin{matrix} x=1 \\ y=-1 \end{matrix} \longrightarrow 1^2-(-1)^2 = 1-1=0$

$x^2-y^2 = (x-y)(x+y)$

$2x+2y = 2(x+y)$

$\dfrac{x^2-y^2}{2x+2y} = \dfrac{(x-y)\cancel{(x+y)}}{2\cancel{(x+y)}} = \dfrac{x-y}{2}$

Se $(x+y)\neq 0 \implies \boxed{\dfrac{x^2-y^2}{2x+2y} = \dfrac{x-y}{2}}$ Vera nel dominio di $\dfrac{x^2-y^2}{2x+2y}$

$x+y=0$

$\dfrac{x^2-y^2}{2x+2y}$ non esiste ma esiste $\dfrac{\cancel{x+y}\,y}{2}$

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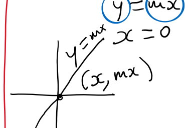
c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^2+y^2}$ $\qquad (0,0) \notin \mathrm{Dom} = \mathbb{R}^2 \setminus \{(0,0)\}$

ma questo limite esiste?

1- Calcolare la funzione ristretta alle rette che passano per il punto $(0,0)$.

$y = mx \longrightarrow$ f ristretta a queste rette è



$f(x,mx) = \dfrac{x^2\cdot(mx)^2}{x^2+(mx)^2} = \dfrac{m^2x^4}{x^2(1+m^2)} = \dfrac{m^2x^2}{(1+m^2)}$

2) $\lim\limits_{x\to 0} f(x,mx) = \lim\limits_{x\to 0} \dfrac{m^2x^2}{1+m^2} = 0$ non dipende dalla retta scelta quindi il limite potrebbe esiste

Se il limite esiste allora vale 0

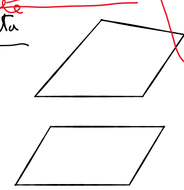
$f(x,y) = \dfrac{x^2y^2}{x^2+y^2} = \boxed{xy}\ \boxed{\left(\dfrac{xy}{x^2+y^2}\right)} \leftarrow$ limitata

$f(x,y) = \dfrac{x}{\sin y}$ $\quad (x,y)\to(0,0)$ NON ESISTE

$y=mx$

$f(x,mx) = \dfrac{x}{\sin mx}$

$\lim\limits_{x\to 0} \dfrac{x}{\sin(mx)} \overset{\text{de l'H.}}{=} \lim\limits_{x\to 0} \dfrac{1}{m\cos(mx)} = \dfrac{1}{m}$



$-\dfrac{1}{2} \leq \dfrac{xy}{x^2+y^2} \leq \dfrac{1}{2}$ $\qquad \left|\dfrac{xy}{x^2+y^2}\right| \leq \dfrac{1}{2}$

usare $\rightarrow$ $\lim\limits_{(x,y)\to(0,0)} xy = 0$ ~~per det. il lim di f.~~

definizione di limite

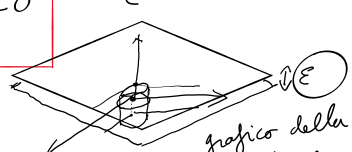
$\forall \varepsilon > 0 \ \exists \delta$ t.c. $\sqrt{x^2+y^2} \leq \delta$

$\Downarrow$

$|xy| \leq \varepsilon$

$|f(x,y) - \ell| = \left|\dfrac{x^2y^2}{x^2+y^2}\right| = \underbrace{|xy|}_{\leq \varepsilon} \underbrace{\left|\dfrac{xy}{x^2+y^2}\right|}_{\leq \frac{1}{2}} \leq \dfrac{\varepsilon}{2}$

$\implies \boxed{\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^2+y^2} = 0}$



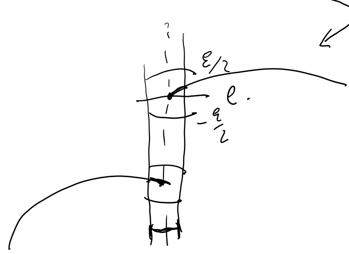


grafico della funzione ristretto al disco di raggio δ sarà compreso tra il piano $z=\frac{\varepsilon}{2}$ e il piano $z=-\frac{\varepsilon}{2}$

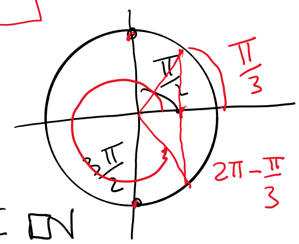
Curve di livello

$f(x,y)=\cos(x^2+y^2)$

$0, \boxed{\frac{1}{2}}, \boxed{\frac{\sqrt{3}}{2}}$

(x,y) t.c. $f(x,y)=c$

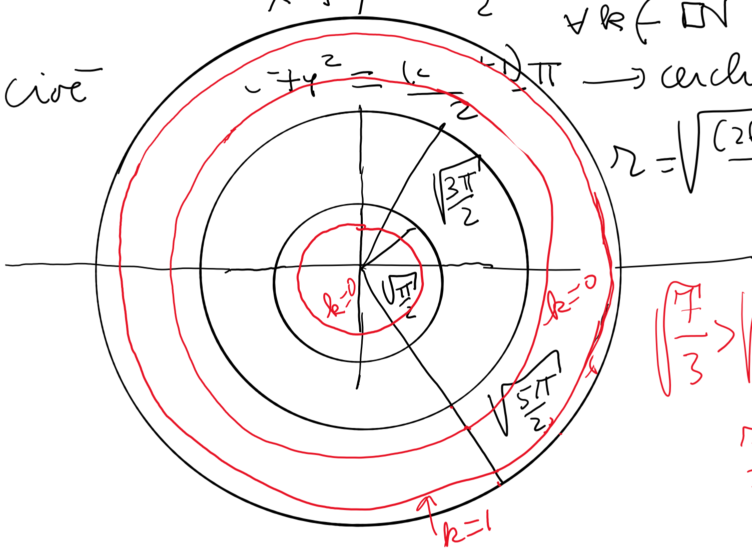
$c=0$ (x,y) t.c. $\cos(x^2+y^2)=0$



$x^2+y^2=\frac{\pi}{2}+k\pi$ $\forall k\in\mathbb{N}$

cioè $x^2+y^2=\frac{(2k+1)\pi}{2}$ → cerchi di raggio

$r=\sqrt{\frac{(2k+1)\pi}{2}}$



$\sqrt{\frac{7}{3}}>\sqrt{\frac{5}{3}}>\sqrt{\frac{3}{2}}$

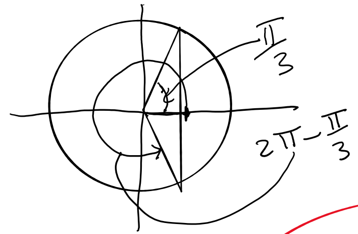
$\frac{7}{3}<\frac{5}{2}$

$\boxed{x^2+y^2=\frac{\pi}{3}+2k\pi}$

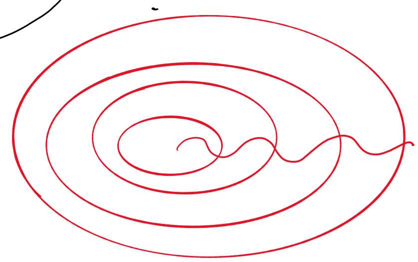
$c = \frac{1}{2}$ $\quad \cos(x^2+y^2) = \frac{1}{2} \Longrightarrow$ $\boxed{\ldots = \frac{\pi}{3} \ldots}$

$\boxed{x^2+y^2 = 2\pi - \frac{\pi}{3}} + 2k\pi$

cerchi di raggio $r = \sqrt{\frac{\pi}{3} + 2k\pi}$ $\quad \forall k \in \mathbb{N}$



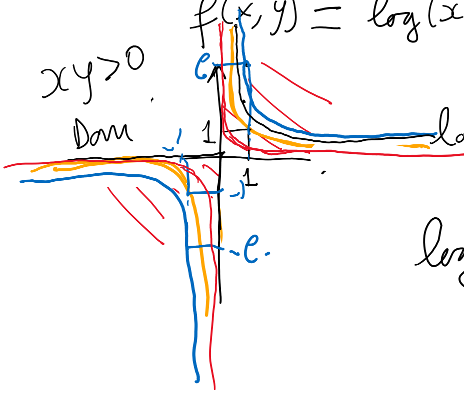
$r = \sqrt{2\pi - \frac{\pi}{3} + 2k\pi}$ $\quad \forall k \in \mathbb{N}$, $k > 1$



$f(x,y) = \log(xy)$ $\qquad \boxed{c = 0}$, $\boxed{c = 1}$, $\boxed{c = -7}$

$xy > 0$

Dom



$\log(xy) = 0 \Longleftrightarrow \boxed{xy = 1}$

$\boxed{y = \frac{1}{x}}$

$\log(xy) = 1 \Longleftrightarrow xy = e$

$\boxed{y = \frac{e}{x}}$

$e \simeq 2{,}71\ldots$

$\log(xy) = -7 \Longleftrightarrow xy = e^{-7} \Rightarrow y = \frac{e^{-7}}{x}$

$f(x,y) = e^{xy}$ $\qquad c = 0,\ c = e,\ c = 4$

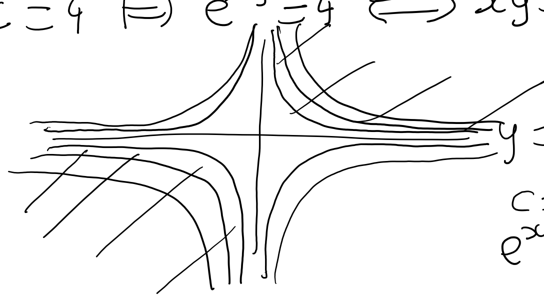
$c = 0 \Longrightarrow (x,y)$ t.c. $\boxed{e^{xy} = 0}$ IMPOSSIBILE $\emptyset$

$e^{xy} > 0$

$c = e \Longrightarrow e^{xy} = e \Longleftrightarrow xy = 1 \qquad y = \frac{1}{x}$

$c = 4 \Longrightarrow e^{xy} = 4 \Longleftrightarrow xy = \log 4 > 1$

$y = \frac{\log 4}{x}$



$y = \frac{1}{x}$

$c = e^{-1}$

$e^{xy} = e^{-1} \Longrightarrow xy = -1$

$y = \frac{-1}{x}$
