Calculus-Unit 1

Applied Computer Science for AI

Blank examination

Voto finale

Postazione:

Cognome:

Nome:

Matricola:

Canale:

Esercizio	Punteggio
1	
2	
3	
4	
Risp. Mult.	
Totale	

Es. 1 [1+2+1 Points] Let $a_n = \frac{n^2+2}{2n^2+1}$ for $n \in \mathbb{N}$

1. Prove that the sequence is bounded by giving an upper bound and a lower for the sequence. (Justify your answer)
2. Find $\lim_{n\to+\infty} a_n$
3. Prove that the sequence is monotone.

Es 2 [3 Points] Prove using induction that for any $n \in \mathbb{N}$ and any $x \in \mathbb{R}$

$$(\sum_{k=0}^{n} x^k)(1-x) = (1-x^{n+1})$$

Es 3 [4 points] Compute the following limit (justify your answer)

$$\lim_{x\to 0^+} \frac{\ln(1+\sqrt[3]{x^2})}{\sqrt{2x}\cdot \sin x}$$

Es 4 [1+2+1+2+1 points] Given the function $f(x) = \frac{x^2}{x-3}$. Determine:
a) Domain
b) Asymptotes
c) Derivative
d) Interval of monotonicity
e) Graph

Es 5 [2 o -1 points] The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^{-x^2}$

(A) Has a minimum and a maximum (B) Has a maximum but no minimum

(C) Has a minimum but no maximum (D) Its minimum is at infinity

Es 6 [2 o -1 punti] The derivative of $f(x) = \sin x \; e^{\cos x}$ is:

(A) $\cos x e^{\cos x}$ (B) $\cos x e^{-\sin x}$ (C) $-\sin^2 x e^{\cos x}$

(D) $e^{\cos x}(\cos^2 x + \cos x - 1)$ (E) None of the previous answers is correct

Es 7 Let $f : [1, 2] \to \mathbb{R}$ differentiable such that $f(1) = 1$, $f(2) = \pi$. Then

(A)[1/2] f is increasing $(1, 2)$ **T** **F** (B)[1/2] $\exists x_o \in (1, 2)$ such that $f'(x_o) = \pi$ **T** **F**

(C)[1/2] f has a maximum and a minimum **T** **F** (D)[1/2] $\exists x_o \in (1, 2)$ su that $f(x_o) = 2$ **T** **F**

Es 8 [2 o -1 punti] $(1 + i)^3$ equals:

(A) $2 + 2i$ (B) $2 - 2i$ (C) $-2i$

(D) $-2 + 2i$ (E) 2

Es 9 [3 o -1 punti] The $\lim_{n \to +\infty} \dfrac{-n^3 + 2n + \ln n^5}{(-1)^n n + 2n^3 + \sqrt{n}}$ equals

(A) 1 (B) $\frac{-1}{2}$ (C) $+\infty$

(D) $-\infty$ (E) The limit does not exist (F) None of the previous answers is correct

Es 10 The function $f : [a, b] \to \mathbb{R}$ is continuous. Say which of the following holds true

(A)[1/2] If $f(a) = f(b)$ then the maximum of f is in (a, b) **T** **F**

(B)[1/2] If $f(\frac{a+b}{2}) = \frac{f(a)+f(b)}{2}$ then f is constant **T** **F**

(C)[1/2] If $f(b) > f(a)$, then f is increasing in (a, b). **T** **F**

(D)[1/2] If $f(x) = 2f(a) + b(x - a)$, then $f(a) = 0$ **T** **F**

(E)[1/2] There exists an x such that $f(x) = \frac{f(a)+f(b)}{2}$ **T** **F**