2nd Italian-Chilean Workshop in PDE's

15th to 19th of January 2018, Roma-Italy

INDAM- Istituto Nazionale di Alta Matematica

Title and abstract of talks

T. Bartsch

A natural constraint approach to normalized solutions of nonlinear Schrödinger equations and systems

The talk will be concerned with the existence of normalized solutions to the system

$$\begin{cases} -\Delta u - \lambda_1 u = \mu_1 u^3 + \beta u v^2 & \text{in } \mathbb{R}^3 \\ -\Delta v - \lambda_2 v = \mu_2 v^3 + \beta u^2 v & \text{in } \mathbb{R}^3 \\ \int_{\mathbb{R}^3} u^2 = a_1^2 \quad \text{and} \quad \int_{\mathbb{R}^3} v^2 = a_2^2 \end{cases}$$

for any $\mu_1, \mu_2, a_1, a_2 > 0$ and $\beta < 0$ prescribed. We present a new approach that is based on the introduction of a natural constraint associated to the problem. Our method can be adapted to scalar nonlinear Schrödinger equations with normalization constraint, and leads to alternative and simplified proofs to some results already available in the literature.

This is joint work with Nicola Soave.

I. Capuzzo Dolcetta

Some results about the weak Maximum Principle

The talk is focused on the validity of the weak Maximum Principle for degenerate elliptic operators on bounded as well as on a class of unbounded domains of the euclidean space. Some of the related papers are:

- I. Capuzzo Dolcetta, F. Leoni and A. Vitolo, The Alexandrov-Bakelman-Pucci weak MaximumPrinciple for fully nonlinear equations in unbounded domains, Commun. Partial Differential Equations 30 (2005)
- H.Berestycki, I. Capuzzo Dolcetta, A. Porretta and L. Rossi, Maximum Principle and generalized principal eigenvalue for degenerate elliptic operators, Journal de Mathematiques Pures et Appliques (2014)
- I. Capuzzo Dolcetta and A. Vitolo, The weak Maximum Principle for degenerate elliptic operators in unbounded domains, International Mathematics Research Notices (2016).

C. Cortázar

Large time behavior of solutions of the porous medium equation in exterior domains

We consider the porous medium equation in the complement of a bounded domain with zero Dirichlet data on its boundary and nonnegative compactly supported integrable initial data.
Kamin and Vázquez, in 1991, studied the large time behavior of solutions of such problem in space dimension 1. Gilding and Goncerzewicz, in 2007, studied this same problem dimension 2. Using their results in the outer field we study the large time behavior of the solution in the near field scale, in particular in bounded sets of the domain.
This a joint work with Fernando Quirós (Universidad Autonoma de Madrid, Spain) and Noemí Wolanski (Universidad de Buenos Aires, Argentina).

G. Dàvila

Existence, nonexistence and multiplicity results for nonlocal Dirichlet problems

We establish various existence, nonexistence and multiplicity results for Dirichlet problems associated to nonlocal Hamilton-Jacobi operators of the form

$$G[u,x] := \inf_{i\in I}\sup_{j\in J}\{L_{ij}(u,x) + b_{ij}(x)\cdot Du(x) + c_{ij}(x)u(x)\}, \tag{1}$$

where L_{ij} is a linear nonlocal operator of order $2s$. This study is accomplished by a careful analysis of the principal eigenvalues of the elliptic operator. Resonance phenomena and anti maximum principles are also established. This is a joint work with A. Quaas (UTFSM) and E. Topp (USACH).

F. De Marchis

Uniqueness result for solutions of Lane Emden problems in convex domains

We consider the question of the uniqueness of the positive solution of the classical Lane Emden equation in bounded domains with Dirichlet boundary conditions. In particular we will focus on convex domains in the plane, in which case we show that, when the exponent of the nonlinearity is large, the solutions satisfying some energy bound are unique. Based on joint papers with M. Grossi, I. Ianni and F. Pacella.

F. Demengel

Ergodic pairs for singular or degenerate fully nonlinear operators

In this talk we will consider the Fully Non linear equations

$$-|\nabla u|^\alpha F(D^2u) + |\nabla u|^\beta + \lambda|u|^\alpha u = f$$

where $\alpha > -1$, $\beta \in]1+\alpha, 2+\alpha]$, F is fully non linear elliptic, $\lambda \geq 0$ and $f \in L^\infty$. I will present some existence results for solutions of the Dirichlet problem when $\lambda \geq 0$ and prove that when λ to zero, the solution either tends to a solution of the same equation the $\lambda = 0$, which is zero on the boundary, or tends to a solution of the ergodic problem

$$-|\nabla u|^\alpha F(D^2u) + |\nabla u|^\beta + c = f, \quad u = +\infty \text{ on } \partial\Omega$$

where c is some constant, said "ergodic". We provide some properties of the ergodic function and ergodic constant, some asymptotic behaviour, and a uniqueness result, extending in that way the well known results of Lasry and PLLions, and of Porretta and Leonori for the p-Laplacian.

Y. Du

Logarithmic shifting in spreading governed by the Fisher-KPP porous medium equation

We consider the large time behaviour of solutions to the porous medium equation with a Fisher-KPP type reaction term and nonnegative, compactly supported initial function in $L^\infty(\mathbb{R}^N)\setminus\{0\}$:

$$(*)\quad u_t = \Delta u^m + u - u^2 \quad \text{in } Q := \mathbb{R}^N \times \mathbb{R}_+, \qquad u(\cdot,0)=u_0 \quad \text{in } \mathbb{R}^N,$$

It is well known that the spatial support of the solution $u(\cdot,t)$ to this problem remains bounded for all time $t>0$. In spatial dimension one it is known that there is a minimal speed $c_*>0$ for which the equation admits a traveling wave solution U_{c_*} with a finite front, and this traveling wave solution is asymptotically stable in the sense that if the initial function $u_0\in L^\infty(\mathbb{R})$ satisfies $\liminf_{x\to-\infty} u_0(x)>0$ and $u_0(x)=0$ for all large x, then $\lim_{t\to\infty}\left\{\sup_{x\in\mathbb{R}}|u(x,t)-U_{c_*}(x-c_*t-x_0)|\right\}=0$ for some $x_0\in\mathbb{R}$. In dimension one we obtain an analogous stability result for the case of compactly supported initial data, not necessarily symmetric. In higher dimensions we show that U_{c_*} is still attractive, albeit that a logarithmic shifting occurs. More precisely, if the initial function in $(*)$ is additionally assumed to be radially symmetric, then there exists a second constant $c^*>0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t\to\infty}\left\{\sup_{x\in R^N}|u(x,t)-U_{c_*}(|x|-c_*t+(N-1)c^*\log t-r_0)|\right\}=0$$

for some $r_0\in\mathbb{R}$ (depending on u_0). If the initial function is not radially symmetric, then there exist $r_1,r_2\in\mathbb{R}$ such that the boundary of the spatial support of the solution $u(\cdot,t)$ for all large time t is contained in the spherical shell $\{x\in\mathbb{R}^N: r_1\le |x|-c_*t+(N-1)c^*\log t\le r_2\}$, and for any $c\in(0,c^*)$, $\lim_{t\to\infty}u(x,t)=1$ uniformly in $\big\{|x|\le c_*t-(N-1)c\log t\big\}$.

This is based on joint work with Fernando Quiros (Madrid) and Maolin Zhou (Armidale).

J. Faya

Towers of nodal bubbles for the Bahri-Coron problem in punctured domains

Let Ω be a smooth bounded domain in $\mathbb{R}^N$ which contains a ball centered at the origin. Consider problem

$$(\wp_\delta)\qquad \begin{cases} -\Delta u = |u|^{2^*-2}u & \text{in } \Omega_\delta, \\ u=0 & \text{on } \partial\Omega_\delta, \end{cases} \tag{2}$$

here $N\ge 3$, $2^*=\frac{2N}{N-2}$ is the critical Sobolev exponent and $\Omega_\delta:=\{x\in\Omega: |x|>\delta\}$. In this talk we will discuss the existence of nodal solutions $(u_{m,\delta})$ for problem $(\wp_\delta)$. Moreover, if Ω is starshaped, we show that the solutions $(u_{m,\delta})$ concentrate and blowup at 0, as $\delta\to 0$, and their limit profile is a tower of nodal bubbles, i.e., they are a sum of rescaled nonradial sign-changing solutions to the limit problem

$$\left\{\ -\Delta u = |u|^{2^*-2}u, \quad u\in D^{1,2}(\mathbb{R}^{\mathbb{N}})\right. \tag{3}$$

centered at the origin.

This is a joint work with M. Clapp and F. Pacella.

G. Galise

A reversed Faber-Krahn type inequality for the truncated Laplacian

We shall consider a family of highly degenerate elliptic operators, arising in geometric context and defined in terms of partial sums of eigenvalues. We shall discuss some very unusual phenomena and their novelty with respect to the uniformly elliptic framework. In particular it will be shown that the symmetry maximizes the principal eigenvalue, at least for square type domains.

This talk is based on a joint work with I. Birindelli and H. Ishii.

M. García-Huidobro

Boundary singularities of positive solutions of quasilinear Hamilton-Jacobi equations

We study the boundary behaviour of the solutions of (E) $-\Delta_p u + |\nabla u|^q = 0$ in a domain $\Omega \subset R^N$, when $N \geq p > q > p-1$. We show the existence of a critical exponent $q_* < p$ such that if $p-1 < q < q_*$ there exist positive solutions of (E) with an isolated singularity on $\partial\Omega$ and that these solutions belong to two different classes of singular solutions. If $q_* \leq q < p$ no such solution exists and actually any boundary isolated singularity of a positive solution of (E) is removable. We prove that all the singular positive solutions are classified according the two types of singular solutions that we have constructed.

M. Grossi

Radial nodal solutions for Moser-Trudinger problems

In this talk we discuss the behaviour of radial nodal solutions of Moser-Trudinger type problems in the ball. Unlike similar perturbative problems we have concentration of the positive part and compactness of the negative one.
This is a joint paper with Daisuke Naimen (Muroran, Japan).

Y. Guo

Multiple solutions for critical quasilinear elliptic equations

We study the existence of infinitely many solutions for the following quasilinear elliptic equations with critical growth:

$$\begin{cases} -\sum_{i,j=1}^N D_j(b_{ij}(v)D_i v) + \frac{1}{2}\sum_{i,j=1}^N b'_{ij}(v)D_i v D_j v = a|v|^{2s-2}v + |v|^{\frac{2sN}{N-2}-2}v & \text{in } \Omega \\ v = 0 & \text{on } \partial\Omega \end{cases} \tag{4}$$

where $b_{ij} \in C^1(\mathbb{R},\mathbb{R})$ satisfies the growth condition $|b_{ij}| \sim |t|^{2s-2}$ at infinity, $s \geq 1$, $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary, a is a constant. Here we use the notations: $D_i = \frac{\partial}{\partial x_i}$, $b'_{ij}(t) = \frac{db_{ij}(t)}{dt}$. In this paper, we will study the effect of the terms $a|v|^{2s-2}v$ and $b_{ij}(v)$ on the existence of an unbounded sequence of solutions for (4). Here, we do not assume the crucialglobal monotone condition. We overcome the difficulties caused by the lack of such monotone condition by performing various kinds of changes of variables. This is the joint work with Yinbin Deng and ShusenYan.

M. Kowalczyk

End-to-end construction of the maximal solution of the Liouville equation

In this paper we consider the Liouville equation $\Delta u + \lambda^2 e^u = 0$ with the Dirichlet boundary conditions in a two dimensional, doubly connected domain Ω. We show that there exists a simple, closed curve $\gamma \in \Omega$ such that for a sequence $\lambda_n \to 0$ there exist a sequence of solutions $u^{max}_{\lambda_n}$ such that $\frac{\lambda_n^2}{\log\frac{1}{\lambda_n}}\int_\Omega e^{u^{max}_{\lambda_n}}\,dx \to c_0|\gamma|$.

F. Leoni

Asymptotic analysis in the ball for almost critical fully nonlinear elliptic equations

We will present recent results about the asymptotic behavior as $\epsilon \to 0$ of the solutions $u_\epsilon > 0$ of the Dirichlet problems

$$\begin{cases} -\mathcal{M}^{\pm}(D^2 u_\epsilon) = u_\epsilon^{p_\pm^* - \epsilon} & \text{in } B_1 \\ u_\epsilon = 0 & \text{on } \partial B_1 \end{cases}$$

where $p_\pm^*$ are the critical (radial) exponents for the Pucci's operators $\mathcal{M}^{\pm}$.

We will show how the solutions u_ϵ concentrate around their maximum point (the origin), while a suitably defined energy associated to the system remains invariant.

A. Malchiodi

On the Sobolev quotient in CR geometry

We consider a class of three-dimensional CR manifolds which are modelled on the Heisenberg group. We give a motion of mass and prove its positivity under the condition that the Webster curvature is positive and that the manifold is embeddable.

We apply this result to the CR Yamabe problem and we discuss the properties of Sobolev-type quotients, which can be in contrast to the Riemannian case. This is joint work with J.H.Cheng and P.Yang.

R. Manasevich

Some quantitative and qualitative results in crime pattern formation

The application of mathematics to crime modeling is a rather new topic which is receiving increasing attention in many places. In this talk we will review some of the important concepts that have originated something that we can call the beginning of a crime science. The subject has many different aspects, some of them give rise to semi-empirical results and some others to results coming from PDE and NLA. We will illustrate these approaches with theoretical and practical results.

R. Mazzeo

Spherical cone metrics on surfaces

There has been a great deal of recent work in the past few years about spherical metrics with prescribed conical singularities, particularly with large cone angles. This includes the substantial contribution by Mondello and Panov. I will describe some new approaches for understanding the space of all such metrics and some new results obtained jointly with Zhu. This includes a new approach to analyzing families of elliptic problems with coalescing singularities.

C. Munoz

Stability and instability of some soliton structures

The purpose of this talk is to describe in some detail the mechanism of stability and instability of some well-known exact solutions for some dispersive models appearing in Physics. Among these mechanisms, we find the standard variational method based in a Lyapunov functional, as well as new dynamical ideas coming from the integrability of the equation. These are recent results obtained in collaboration with Alejo (Florianopolis, Brazil) and Palacios (U. of Chile), concerning solutions of the modified KdV, Nonlinear Schrödinger, and sine-Gordon equation in the standard energy space.

M. Musso

Singularity formation in critical parabolic equations

In this talk I will discuss some recent constructions of blow-up solutions for a Fujita type problem for power p related to the critical Sobolev exponent. Both finite type blow-up (of type II) and infinite time blow-up are considered. This research program is in collaboration with C. Cortazar, M. del Pino and J. Wei.

F. Pacella

Overdetermined problems and constant mean curvature surfaces in cones

We present some recent results about:

i) characterization of domains in cones which admit a solution of a partial overdetermined problem of Serrin type;

ii) characterization of constant mean curvature surfaces with boundary in cones.

As in the classical case of surfaces without boundary, the above questions are strictly related and we show that the domains and surfaces corresponding to the above questions are spherical sectors and spherical caps (respectively). Finally, connections with a relative isoperimetric inequality in cones proved in [Lions-Pacella, 1990] will be emphasized. These results have been obtained in collaboration with G.Tralli.

S. Patrizi

Regularity of interfaces for a Pucci type like segregation problem

We consider free boundary problems involving fully nonlinear extremal operators. Model examples are limit profiles of segregation problems. We study the regularity of the free boundary. More precisely, we characterize the set of regular points and we show that it is an open subset of the free boundary locally of class $C^{1,\alpha}$. This is a joint paper with Luis Caffarelli, Véronica Quitalo and Monica Torres.

K. Payne

Principal eigenvalues for k-Hessian operators

For fully nonlinear k-Hessian operators on bounded strictly $(k-1)$-convex domains of Euclidean space, a characterization of the principal eigenvalue associated to a k-convex and negative principal eigenfunction will be given as the supremum over values of a spectral parameter for which admissible viscosity supersolutions obey a minimum principle. The admissibility condition is phrased in terms of elliptic branches in the sense of Krylov [Trans. Amer. Math. Soc. 1995] that correspond to selecting k-convex functions. Moreover, the associated principal eigenfunction is constructed by an iterative viscosity solution technique, which exploits a compactness property coming from the establishment of a global Hölder continuity property for the approximating equations. This is joint work with Isabeau Birindelli.

A. Quaas

TBA

A. Rodríguez

Loss of boundary conditions for two nonlinear parabolic equations dominating gradient terms

We present a contribution to the study of qualitative properties of viscosity solutions of nonlinear parabolic equations whose rate of growth with respect to the gradient variable makes the corresponding term the dominant one in the equation. Specifically, we show that the phenomenon of *loss of boundary conditions* (LOBC, for short) occurs for two model problems with prescribed boundary and initial value data.

The first is the fully nonlinear case:

$$u_t - \mathcal{M}^-(D^2u) = |Du|^p \quad \text{in } \Omega \times (0,T), \tag{5}$$

$$u|_{\partial\Omega\times(0,T)} = 0, \qquad u(\cdot,0) = u_0 \in C^1(\overline{\Omega}). \tag{6}$$

Here $\Omega \subset \mathbb{R}^N$ is a bounded, smooth domain, $T > 0$; $\mathcal{M}^-$ denotes Pucci's (minimal) extremal operator, $\mathcal{M}^-(X) = \inf\{\mathrm{tr}(AX) | \lambda \mathrm{Id} \leq A \leq \Lambda \mathrm{Id}\}$, where A and X are symmetric $N \times N$ matrices and $0 < \lambda \leq \Lambda$; and $p > 2$. The second model problem is of nonlocal character:

$$u_t + (-\Delta)^s u = |Du|^p \quad \text{in } \Omega \times (0,T), \tag{7}$$

$$u|_{\mathbb{R}^N\setminus\Omega\times(0,T)} = 0, \qquad u(\cdot,0) = u_0 \in C^\beta(\overline{\Omega}), \tag{8}$$

where Ω and T are as before, $(-\Delta)^s$ denotes the well-known fractional Laplacian operator with $s \in (0,1)$, p satisfies

$$s + 1 < p < \frac{s}{1-s}, \tag{9}$$

and $\beta > {}^{p-2s}/_{p-1}$; (9) restricts the value of s to $(0.618\ldots, 1)$, where $0.618\ldots$ is the constant sometimes called *reciprocal golden ratio*. These restrictions, however, are related to our methods and may not be essential.

For each of our model problems, we prove that *a)* there exists a small time $T^* > 0$ depending only on the initial condition u_0 (specifically, on $\|u_0\|_{C^1(\overline{\Omega})}$ and $\|u_0\|_{C^\beta(\Omega)}$, respectively) and universal constants, such that the corresponding viscosity solution satisfies the boundary data in the classical sense (pointwise); and *b)* LOBC occurs depending on a largeness condition for u_0 given in terms of an eigenfunction of $\mathcal{M}^-$ and $(-\Delta)^s$, respectively. Joint work with Alexander Quaas.

B. Ruf

A heat equation with exponential nonlinearity and with singular data in $\mathbb{R}^2$

We consider a semilinear heat equation with exponential nonlinearities and singular data in $\mathbb{R}^2$.

In $\mathbb{R}^N$, $N \geq 3$, critical growth related to singular initial data is polynomial and has been studied by several authors. Existence and non-existence results for singular initial data in suitable L^p-spaces were obtained by F. Weissler and H. Brezis - T. Cazenave; furthermore, non-uniqueness results for certain singular initial data were given by W.-M. Ni - P. Sacks and E. Terraneo.

In $N = 2$ critical growth is given by nonlinearities of exponential type (cf. N. Trudinger - J. Moser). With prove that similar phenomena, namely existence, non-existence and non-uniqueness, occur for suitable exponential nonlinearities and singular initial data in certain Orlicz spaces.

This is joint work with N. Ioku (Ehime University) and E. Terraneo (University of Milan).

M. Saez

On the uniqueness of graphical mean curvature flow

In this talk I will discuss recent work with P. Daskalopoulos on sufficient conditions to prove uniqueness of complete graphs evolving by mean curvature flow, when the behavior at infinity is not prescribed.

S. Salsa

Regularity of higher order in two-phase free boundary problems

We present new results on the regularity for free boundary problems governed by uniformly elliptic equations with distributed sources. Joint work with Daniela de Silva and Fausto Ferrari.

P. Sicbaldi

Some advances on the geometry of overdetermined elliptic problems.

In this talk I will discuss the geometry of overdetermined elliptic systems such as

$$\begin{cases} \Delta u + f(u) = 0 & \text{in} \quad \Omega \subset \mathbb{R}^n,\ n \geq 2 \\ u = 0 & \text{on} \quad \partial\Omega \\ \frac{\partial u}{\partial \vec{n}} = \text{constant} & \text{on} \quad \partial\Omega \end{cases} \tag{10}$$

I will present some recent rigidity results, I will construct new unexpected solutions for some nonlinearities f, and I will prove some geometric properties.

B. Sirakov

Optimal exponent in the boundary weak Harnack inequality for elliptic operators in divergence form

We review some recent results on boundary (weak) Harnack inequalities for uniformly elliptic divergence and non-divergence form operators. In the divergence case we find the optimal L^ε-norm in which the fraction of a positive supersolution of an elliptic equation and the distance to the boundary of a given domain is bounded. We present some applications to solvability and uniform a priori bounds for the Dirichlet problem.

Y. Sire

Some results on fractional harmonic maps

In a recent series of papers, Da Lio and Rivière introduced the concept of fractional harmonic map to understand the Wente inequality in one dimension. We will report on some results where we study on one hand the Ginzburg-Landau approximation of half-harmonic maps into spheres and on the other hand the regularity in one dimension of super-critical fractional harmonic maps. We will also introduce a flow of such maps into spheres and homogeneous spaces and study global weak solutions.

S. Terracini

The nodal set of solutions to sublinear equations

We are concerned with the nodal set of solutions to equations of the form

$$-\Delta u = \lambda_+ \left(u^+\right)^{q-1} - \lambda_- \left(u^-\right)^{q-1} \quad \text{in } B_1$$

where $\lambda_+, \lambda_- > 0$, $q \in (0,2)$, $B_1 = B_1(0)$ is the unit ball in $\mathbb{R}^N$, $N \geq 2$, and $u^+ := \max\{u, 0\}$, $u^- := \max\{-u, 0\}$ are the positive and the negative part of u, respectively. This class includes, the two-phases *unstable obstacle problem ($q = 1$) as well as singular equations.* Notice that the right hand side is not locally Lipschitz continuous as function of u, and precisely has sublinear character for $1 < q < 2$, and a discontinuos character for $q \leq 1$.

We consider the two phases problem treating simultaneously the cases $q =\leq$ and $1 < q < 2$, proving the following main results: (a) an a priori $\mathcal{C}^{1,\alpha}$ estimate on the solutions; (b) the finiteness of the vanishing order at every point and the complete characterization of the order spectrum; (c) a weak non-degeneracy property; (d) regularity of the nodal set of any solution: the nodal set is a locally finite collection of regular codimension one manifolds up to a residual singular set having Hausdorff dimension at most $N-2$ (locally finite when $N=2$) and a partial stratification theorem.

Ultimately, the main features of the nodal set are strictly related with those of the solutions to linear (or superlinear) equations, with two remarkable differences. First of all, the admissible vanishing orders are can not exceed the critical value $2/(2-q)$. Moreover, at the threshold $2/(2-q)$, we find a multiplicity of homogeneous solutions, yielding the *non-validity* of any estimate of the $(N-1)$-dimensional measure of the nodal set of a solution in terms of the vanishing order.

The proofs are based on Almgren's and Weiss type monotonicity formulæ, blow-up arguments and the classification of homogenous solutions.

This is a joint work with Nicola Soave.

E. Topp

Regularity results for nonlocal Hamilton-Jacobi equations and application to periodic homogenization

In this talk we report Lipschitz estimates for bounded solutions to nonlocal, degenerate elliptic Hamilton-Jacobi equations with coercive gradient terms. These regularity results are obtained via Ishii-Lions method, and are used in the study of periodic homogenization of associated parabolic equations.

Joint work with Guy Barles - Olivier Ley; and Annalisa Cesaroni - Martino Bardi.

Z. Q. Wang

A class of quasilinear elliptic equations via a regularization approach

For a class of quasilinear elliptic problems including the modified nonlinear Schrödinger type equations, which lack both smoothness and compactness in the variational formulations, we develop a regularization approach for the existence and multiplicity theory. This allows us to overcome difficulties of both smoothness and compactness issues involved. By establishing convergence results we obtain existence and multiplicity results for the quasilinear equations.

S. Yan

Local Uniqueness for Bubbling Solutions

In this talk, I will present some recent results on the local uniqueness for bubbling solutions for some elliptic problems.