A life with Algebra

Claudio Procesi.

Conference for my 70th birthday, Roma 9 June 2011

(ロ)、

<ロト <回ト < 注ト < 注ト

Laurea Roma, 1963, Ph. D. Chicago 1966



I was a student of HERSTEIN, Chicago 1987



My work

I count 80 published research papers, plus 5 preprints of present research, 3 books, 5 lecture notes, 14 communications or seminars for a total of 107

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Non commutative Algebra

I started with Non commutative Algebra.

In this field I have written 25 papers and one book, plus 5 papers and a lecture note on *quantum groups*

Non commutative Algebra

In 1964–65 there was a special year on Algebra in Chicago, with Amitsur, Paul Cohn, Jacobson and many others, there I started to learn *Rings with polynomial identities*

3 theorems impressed me most

- the Amitsur–Levitzki identitity $\sum_{\sigma \in S_{2n}} sign(\sigma) x_{\sigma(1)} \cdots x_{\sigma(2n)} = 0 \text{ for } n \times n \text{ matrices.}$
- **②** The Theorem of Amitsur that the free algebra modulo the identities of $n \times n$ matrices is a domain order in a division algebra of order n^2 over its center.
- The Theorem of Kaplansky that primitive PI algebras are finite matrices.

My Ph. D. program

These Theorems suggested the possibility of doing some kind of *non–commutative affine algebraic geometry* where points have coordinates *matrices* and equations are non–commutative.

A first encounter with invariants

In developing this program I met naturally *invariants of matrices* as follows. If X is a 2×2 matrix the Cayley–Hamilton theorem gives

$$X^2 - tr(X)X + \det(X) = 0$$

If Y is another matrix you get thus

$$[X^2, Y] = tr(X)[X, Y] \implies Tr(X) = [X^2, Y][X, Y]^{-1}$$

the trace is a non-commutative rational function! So is the determinant!

$$\mathsf{det}(X) = [X^2Y, XY][Y, XY]^{-1}$$

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

A first encounter with invariants

So, Amitsur had introduced the division algebra D(m; n) of rational functions in *m*-matrix variables $(m \ge 2)$ and I discovered that its center is the field of invariants under conjugation of the space of *m*-tuples of matrices. In order to do this I introduced the ring of generic matrices.

Hilbert's 17th problem, a non commutative version

I took serioulsy working with rational functions of matrices so I asked if a symmetric positive rational function is always a sum of functions $F(X)F(X)^t$. With Murray Schacher we proved a weaker result, and only recently it was shown that the stronger conjecture

(日)



is false.

Il calcolo letterale

I took serioulsy working with rational functions of matrices but we really do not have a satisfactory theory of normal forms as for commutative variables, the only complete case is for 2×2 matrices, which I proved using invariants and standard monomials. My paper Computing with 2×2 matrices was rejected by Inventiones.

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

A first encounter with *invariants*

At that time *Invariant Theory* was knowing a big revival and people were reading again the fundamental papers of Hilbert.

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Mike Artin

Mike Artin read my thesis and developed a beautiful characterization of Azumaya algebras by PI theory, at the same time he used Hilbert's invariant theory to study semisimple representations of algebras, since they correspond to *closed orbits*.



Mike Artin

also asked if invariants om *m*-tuples of matrices were generated by *traces of monomials* i.e. elements $tr(X_{i_1}X_{i_2}...X_{i_m})$,

we did not realize that

there was a paper by Sibirskii (1968) and a large literature by authors working on continuum mechanics on this topic (Rivlin, Spencer, Smith).

I never understood if one could consider this statement known to, say, Gordan!!

Invariants of matrices

nevertheless I wrote a big paper on Invariants of matrices and discovered that all relations could be derived from the Cayley–Hamilton identity. This was proved independently by Razmyslov who also proved the best estimates for bounding the



▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

degrees of the generators of invariants.

Invariants of matrices

It took me several years to fully understand that,

when computing with matrices, the operation of trace should be included!

in 1987 I presented to Herstein (the last time I saw him before his death in 1988) one of my best results in non commutative algebra: "A formal inverse to the Cayley–Hamilton Theorem"

Positive characteristic

I started to see if, by pure arguments of non commutative algebra one could develop the Theory in all characteristics, where a priori it was unclear if Hilbert's theory could apply. In particular I showed that the algebra of invariants of matrices is finitely generated (now a consequence of general theory). Finite generation of invariants is *Hilbert's* 14th problem

Hilbert's 14th problem

Is the ring of invariants of an algebraic group acting on a polynomial ring always finitely generated? No, a counterexample was constructed by Masayoshi Nagata in

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



1959.

Geometric invariant theory

(or GIT) is a method for constructing quotients by group actions in algebraic geometry, used to construct moduli spaces. It was developed by David Mumford in 1965, using ideas from the paper



・ロト ・四ト ・ヨト ・ヨト ・ヨ

(Hilbert 1893) in classical invariant theory.

Mumford's conjecture

In order to apply the theory of Hilbert in positive characteristic Nagata and Mumford understood that the basic property is geometric reductivity so Mumford conjectured that reductive groups (as for instance the general linear group) are geometrically reductive

▲ロ ▶ ▲周 ▶ ▲ 国 ▶ ▲ 国 ▶ ● の Q @

Weyl–Schur duality

I discovered Weyl's book, the classical groups and the many facets of the interplay between invariants and representations, as the Theory of Schur, so I thought that one could try to prove Mumford's conjecture for the linear group GL(V) by showing that the group algebra of the symmetric group S_{∞} in the limit of the actions of S_m on $V^{\otimes m}$ is in some weak sense semisimple (no nil ideals).

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Formanek (1976) was visiting Pisa

and told be that he had proved that $F[S_{\infty}]$ has no nil ideals (not what I wanted), so we adapted his theorem and obtained Mumford's conjecture for classical groups. The general Theorem was obtained at the same time by structure Theory by Haboush.

・ロト・日本・モート ヨー うくつ



Characteristic free invariant theory

The time was ripe to try to understand how much of Weyl's book would carry over to finite characteristic. Several inputs came in this direction and also my collaboration with Corrado De Concini started on this problem. On one hand I listened to a talk by Giancarlo Rota who had just developed the *Theory of double standard tableaux*, then we discovered a paper of Igusa on projective normality of the Grassmann variety via ideas on tableaux



due to Hodge.

Characteristic free invariant theory

With Corrado we developed a fair amount of this theory using a mix of geometry and combinatorics. Then this theory led us to a deep exchange with Seshadri and his school. Seshadri was generalizing the Theory of Hodge from a completely different (more geometric) viewpoint.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



Characteristic free invariant theory

In a different direction this led us to study a general theory of invariant ideals (generalizing determinantal ideals) with Eisenbud, this was a germ for a very long different trip into *enumerative geometry*

(日) (四) (日) (日) (日)



Inequalities

My last paper, properly on invariant theory, was with Gerald Schwarz,



this put us in an interesting (but puzzling) contact with the physicists who were trying to understand *grand unification* of forces.

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Matrices and singularities

before plunging into *enumerative geometry* I should recall the very fruitful collaboration with Hanspeter Kraft and the deep theory of singularities for closures of conjugacy classes that we developed, by a mix of geometry, invariant theory and representation theory.



◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ● ● ● ● ●

Those were exciting years.

Hilbert's 15th problem and Enumerative geometry

asks for a Rigorous foundation of Schubert's enumerative calculus, we were puzzled when we learned that *There are 3264 conics tangent to 5 generic conics* Chasles 1864. I remember listening to a lecture by Israel Vaisencher who explained the beautiful theory of *complete conics* which lies behind this computation.



Hilbert's 15th problem

More or less at the same time Bill Fulton gave some lectures in Cortona on the topic and I recalled a weird conversation with George Kempf on complete linear systems.

▲□▶ ▲□▶ ▲三▶ ▲三▶ 三三 のへの



Hilbert's 15th problem

Maybe Corrado remembers different inputs, anyway we started to try to understand if there was a general Theory behind.

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Hilbert's 15th problem

We discovered papers by Semple and Tyrell who did very explicit matrix computations.

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Equivariant embeddings

At the same time it was available the Theory of *torus embeddings* on one hand, on the other Luna and Vust had started a very general Theory.

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Equivariant embeddings

Finally Demazure had developed a theory of degenerations of symmetric spaces for the case of classical groups.

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Wonderful embeddings

With Corrado we were able to generalize the methods of Semple and Tyrell and develop a general construction of a *wonderful model* for symmetric varieties which included also Demazure's models. We then were able to transform Kleiman's theory into a general theory of *Halphen rings*.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



Springer's Theory

The 80's were years in which deep connections between representation theory and geometry were discovered and it was natural to look at some of these aspects, in particular with Corrado we solved a beautiful conjecture of Kraft and then George Lusztig visited Roma for a year (1987?), we wrote a very difficult paper together with Corrado, at times we despaired to be able to finish it and I think it was only a very odd combination of rather different ways of thinking Mathematics which allowed us to be successful.



Quantum groups

I was not eager to work on Quantum groups but was dragged by Corrado who had written an important paper with Victor Kac. We wrote some very complicated and technically extremely heavy papers, and I am not sure if anybody read them! Then we tried to understand the point of view of Drinfeld and discovered the beautiful theory of Kohno–Drinfeld and the connection,



via the work of Vassiliev and Kontsevich, with knot theory.

Knots and braids

With the experience on wonderful models we developed a very combinatorial and geometric theory aimed at replacing configurations of hyperplanes with normal crossing singularities. We then applied the Theory to the ideas of Drinfeld and Kontsevich on the Kniznik–Zomolodchikov equation and the universal Vassiliev invariant.

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

Knots and braids

Later on Corrado started a fruitful collaboration with Salvetti and again dragged me into the game. We wrote a mysterious paper on the equation of degree 6, a *tour de force* in homotopy theory and obstruction theory (something I missed to learn as a student in Chicago).

<u>n</u>!

Here I acted mostly as a catalyzer





<ロト <回ト < 三ト < 三ト = 三

hyperplane arrangements, polytopes and box-splines

we arrived to this naturally from our theory of wonderful models trying to understand a discussion with Michele Vergne. It was very curious to enter in the world of splines and numerical analysis

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



Index and splines, The REAL SURPRISE

was to discover the applications to index of transversally elliptic operators and equivariant K-theory, the theory of Dahmen-Micchelli from numerical analysis was the *perfect fit* for index theory

▲□▶ ▲□▶ ▲三▶ ▲三▶ - 三 - のへで



Non com. Algebra Invariants Enumerative geometry Springer's Theory Quantum groups Knots n! Index and splines PDE's 1

Murphy's law or a Variation of Peter Principle

"In Research Every (ambitious) Scientist Tends to Rise to His Level of Incompetence."

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Non linear PDE's

My last adventure has been in PDE, and the responsible is my daugther Michela who started to ask me a seemingly innocuous problem on Euclidean Geometry and then dragged me into the mysteries of KAM theory.

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

The culprit



◆□▶ ◆□▶ ◆目▶ ◆目▶ ▲□▶ ◆□◆



 Sopra un teorema di Goldie riguardante la struttura degli anelli primi con condizioni di massimo.
 Atti Accad. Naz. Lincei (8) 34 (1963)
 Sugli anelli principali ed un teorema di Goldie.
 Atti Accad.Naz. Lincei (8) 36 (1964)
 (with L. Small) On a theorem of Goldie.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Journal of Algebra v. 2 3, pp.80-84 (1965)



- 4 Su un teorema di Faith-Utumi.
- Rend. Mat. Pura e Appl. (5) 24 (1965)
- 5 (with S. A. Amitsur) Jacobson rings and Hilbert algebras with polynomial identities.

- Ann. Mat. Pura e Appl. (4) 71 (1966)
- 6 The Burnside problem.
- Journal of Algebra 4 3, pp. 421-425 (1966)



- 7 Non commutative Jacobson rings. Ann. Scuola Norm. Sup. Pisa (3) 21 (1967)
- 8 Non commutative affine rings. Atti Accad. Naz. Lincei Memorie (8) 8 (1967)
- 9 (with L. Small) Endomorphism rings of modules over PI-algebras. Mathematics Zeit. 106 (1968)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00



10 Sugli anelli non commutativi zero dimensionale con identitĹ polinomiale.

```
Rend. Circ. Mat. Palermo (2) 17, pp. 5-12 (1968)
```

11 Sulle identitL delle algebre semplici.

```
Rend. Circ. Mat. Palermo (2) 17 (1968)
```

12 A non commutative Hilbert Nullstellensatz. Rend. Mat. e appl. (5) 25 (1966)



- 13 On the identities of Azumaya algebras. Ring theory. Acad.Press,pp. 287-295 (1972)
- 14 Dipendenza integrale nelle algebre non commutative. Symposia Mathematica v. VIII Acad. Press, pp. 295-308 (1972).

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

- 15 On a theorem of M. Artin.
- Journal of Algebra v.22 2, pp. 309-315 (1972)



16 Rings with polynomial identities.

Pure and Applied Mathematics v. 17, M.Dekker (1973)

17 Sulle rappresentazioni degli anelli e loro invarianti.

Symposia Mathematics v.XI Acad. Press, pp. 143-159 (1973)

18 Finite dimensional representations of algebras.

Israel Journal of Mathematics 19 (1974)



19 (with I.N. Herstein and M. Schacher) Algebraic valued functions on non commutative rings.

Journal of Algebra 36 1, pp. 128-150 (1975)

20 (with E. Formanek) Mumford's conjecture for the general linear group.

Advances in Mathematics 19 3, pp. 292-305 (1976)

21 Central polynomials and finite dimensional representations of rings.

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

Atas Esc. Algebra I.M.P.A. pp. 2-31 (1973)



22 (with M. Schacher) A non commutative real Nullstellensatz and Hilbert's 17th problem.

Ann. of Mathematics (2) 104 (1976)

23 The invariants of *n*?*n* matrices.

Bull. Am. Mathematics Soc. 82 6, pp. 891-892 (1976)

24 The invariant theory of *n*?*n* matrices.

Advances in Mathematics 19 (1976)



25 (with C. De Concini) A characteristic free approach to invariant theory.

```
Advances in Mathematics 21 (1976)
```

26 Les base de Hodge dans la theorie des invariants. Séminaire d'algébre P. Dubreil. Lecture Notesin Mathematics 641, Springer (1978)

27 Positive symmetric functions.

Advances in Mathematics 29 2, pp.219-225(1978)



- 28 (with H. Kraft) Classi coniugate in GL(n,C).
- Rend. Sem. Mat. Padova 59. pp.209-222 (1978)
- 29 (with H. Kraft) Closures of conjugacy classes of matrices are normal.
- Inv.Math. 53, pp.227-247 (1979)
- 30 Trace identities and standard diagrams.
- Ring theory. Lecture notes in Pure and Appl. Mathematics 51 M. Dekker, pp. 191-218 (1979)

Non com. Algebra Invariants Enumerative geometry Springer's Theory Quantum groups Knots n! Index and splines PDE's N



31 Sulla formula di Gordan Capelli. Univ. Ferrara (1979)
32 Invariante. Voce Enciclopedia, VII. Einaudi pp.891-949 (1979)
33 Young diagrams, standard monomials and invariant theory.
Proc. I.C.M. Helsinki. Acad. Sci. Finnica, pp.537-542(1980)



34 (with C. De Concini and D.Eisenbud) Young diagrams and determinantal varieties.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Inv. Mathematicae 56, pp.129-165 (1980)

35 Algebre cicliche e problema di Luroth.

B.U.M.I. (5) 18-A, pp. 1-10 (1981).

36 (with H. Kraft) Minimal singularities in GL_n . Inv. Mathematicae 62, pp.503-515 (1981)



- 37 (with C. De Concini) Symmetric functions, conjugacy classes and the Flag variety.
- Inv. Mathematicae 64, 203-219 (1981)
- 38 (with C. De Concini e D. Eisenbud) Hodge Algebras. Astérisque 91 (1982)
- 39 (with H. Kraft) On the geometry of conjugacy classes in classical groups.
- Commentarii Mathematici Helvetici 57, pp. 539-602 (1982)



- 40 A primer of invariant theory. (Note di G.Boffi). Brandeis Lecture Notes 1, (1982)
- 41 (with C. De Concini) Complete Symmetric Varieties.
 C.I.M.E. 1982, Springer Lecture Notes 997, pp. 1-44 (1983)
 42 Computing with 2?2 matrices.
 Journal of Algebra, v.87,n. 2, 342-359 (1984)

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



43 (with C. De Concini) Complete Symmetric Varieties II (Intersection Theory).

Advanced Studies in Pure Mathematics 6, Algebraic groups and related topics pp.481-513 (1985)

44 (with G. Schwarz) Inequalities defining orbit spaces. Inv. Math.v. 81, 3,pp. 539-554 (1985)

45 (with G. Schwarz) The geometry of orbit spaces and gauge symmetry breaking in supersymmetric gauge theories. Physics Letters B (1985)

My papers

- 46 (with C. De Concini) Cohomology of compactifications of algebraic groups.
- Duke Mathematical Journal 58, pp. 585-594, (1986)
- 47 A formal inverse to the Cayley Hamilton theorem. Journal of Algebra 107 1, pp.63-74 (1987).
- 48 (with L. Le Bruyn) Étale local structure of matrix invariants and concomitants.

Algebraic groups Utrecht 1986, Springer Lecture Notes 1271,pp.143-175 (1987)



49 (with H. Kraft) Graded morphisms of, G-modules.
Ann. Inst. Fourier T. XXXVII 4, pp. 161-166 (1987).
50 (with G. Schwarz) Defining orbit spaces by inequalities.
Banach Center Publications, v. 20, pp. 365-372 (1988)
51 (with C. De Concini, M. Goresky, R. Mac Pherson) On the geometry of quadrics and their degenerations.
Comm.Mathematics Helvetici 63, pp. 337-413 (1988)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00



52 (with J. L. Loday) Homology of symplectic and orthogonal algebras.

Advances in Mathematics, v. 69 1, pp. 93-108 (1988)

53 (with C. De Concini, G. Lusztig) Homology of the zero set of a nilpotent vector field on a flag manifold.

Journal of A.M.S.,v. 1, pp. 15-34 (1988)

54 Imagination and the building of Mathematics, Lexicon Philosophicum 3,pp.1-4 (1988)



- 55 L'immaginario matematico, problemi classici e congetture. Fondamenti 10, pp. 77-93 (1988)
- 56 (with E. Arbarello, C. De Concini, V. Kac) Moduli Spaces of Curves and Representation Theory.
- Comm. Math.Phys. 117, pp. 1-36 (1988)
- 57 (with J. L. Loday) Cyclic homology and Lambda-operations. Algebraic K-Theory: Connections with Geometry and Topology. NATO ASI Series C, Vol. 279, pp.209-224 (1989).

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00



58 (with H. Kraft) A special decomposition of the nilpotent cone of a classical Lie algebra.
Astérisque 173-74, pp. 271-279 (1989)
59 (with L. Le Bruyn) Semisimple representations of quivers.

T.A.M.S., v. 317 2, pp. 585-598 (1990)

60 (with B, Tirozzi) Metastable states in the Hopfield model. Int.Journal of Modern Physics B, v. 4,1, pp. 143-150 (1990)

My papers

- 61 The toric variety associated to Weyl chambers Mots, Hermes (1990)
- 62 (with E. Bifet, C. De Concini) Cohomology of regular embeddings.
- Advances in Mathematics, v. 82, n. 1, pp.1-34 (1990)
- 63 (with P. Littelmann) Equivariant cohomology of wonderful compactifications.
- Operator algebras, Unitary Representations, Enveloping Algebras, and Invariant Theory. Progress in Mathematics Vol. 92, BirkhŁuser, pp. 219-262 (1990)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

My papers

- 64 (with P. Littelmann) On the Poincaré Series of Invariants of Binary forms.
- Journal of Algebra, V. 133 2, pp. 490-499 (1990)
- 65 (with M. Brion) Action d'un tore dans une variété projective. Operator algebras, Unitary Representations, Enveloping Algebras, and Invariant Theory. Progress in Mathematics Vol. 92, BirkhŁuser, pp. 509-539 (1990)
- 66 (with Xambo) On Halphen's first formula (Zeuthen Colloquium edited by S.Kleiman),

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Contemporary Mathematics, v.123, pp. 199-211 (1991)



67 Aspetti geometrici e combinatori della teoria delle rappresentazioni del gruppo unitario (notes by Rogora), Quaderni U.M.I. 36 (1991)
68 (with A. Garsia) On certain graded S_n-modules and the

q-Kostka polynomials,

Advances in Math. (1992)

69 (with C.De Concini, V. Kac) Quantum coadjoint action, Journal of A.M.S.,v.5,n.1,pp.151-189 (1992)



- 70 (with De Mari,Shayman) Hessenberg varieties, T.A.M.S. v.332,n.2,pp.529-534 (1992)
- 71 (with E. Formanek) The Automorphism Group of a Free Group is not linear.
- Journal of Algebra v.149, n.2 pp.494-499 (1992)
- 72 (with De Concini), Quantum groups (C.I.M.E. notes) Springer L.N.M.1565 (1993)

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

My papers

- 73 (with L.Small, E.Kirkman) A q-analogue of the Virasoro algebra Comm. in Algebra 22 (10), 3755-3774 (1994)
- 74 (with C. De Concini, V.Kac) Some remarkable degenerations of quantum groups.
- Comm. Math. Phys., 157, no. 2, 405-427 (1993)
- 75 (with C. De Concini, V. Kac) Some quantum analogues of solvable Lie groups
- "Geometry and analysis" Tata Institute of Fundamental Research, pp.41-65 (1995)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●



76 (with De Concini) Wonderful models of subspace arrangements Selecta Mathematica, New Series Vol. 1, No. 3 pp. 459-494 (1995)

77 (with De Concini) Hyperplane arrangements and holonomy equations

Selecta Mathematica, New Series Vol. 1, No. 3 pp. 495-535 (1995)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

78 (with R. Alperin) On the Gassner representation J.Algebra, 181, 16-25 (1996)



- 79 (with De Concini) Quantum Schubert cells Journal of Australian Math. soc. "Algebraic groups and Lie groups", ed. Lehrer, pp. 127-160, (1997)
- 80 Deformations of representations
- Methods in ring theory (Levico Terme, 1997), 247–276, Lecture Notes in Pure and Appl. Math., 198, Dekker, New York, (1998)
- 81 (with MacPherson) Making conical stratifications into wonderful ones,
- Selecta Mathematica, New Series Vol. 4, pp. 125-139 (1998)



82 (with P. Papi) Invarianti dei Nodi
Quaderni dell'Unione Matematica Italiana n.45 (1998)
83 (with G. Bini, C. De Concini, M. Polito) On the work of Givental relative to mirror symmetry
Appunti Scuola Normale Superiore (1998)
84 (with S. Fomin) Fibered quadratic Hopf algebras related to Schubert calculus
Journal of Algebra 230, 174-183 (2000)

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



- 85 150 years of Invariant Theory, Israel Mathematical Conference Proceedings
- Vol. 12, pp. 5-21 (1999)
- 86 (with De Concini, M. Salvetti) Arithmetic properties of the cohomology of braid groups Topology 40, no. 4, 739–751 (2001)

87 (with De Concini, M. Salvetti, Stumbo) Arithmetic properties of the cohomology of Artin groups Annali Scuola Normale, s.IV, Vol. XXVIII, Fasc. 4 (1999)



88 Enumerative geometry from the Greeks to Strings, Mathematics and the 21st century,
Eds. A.A. Ashour and A.S. F. Obada. World Scientific Publishing pp. 59-67 (2001)
89 On the n! conjecture
Séminaire Bourbaki 2001/2002. (Astérisque, vol. 290, Exposé 898) (2002)
90 De Concini, C.;Procesi, C., Salvetti M. (2004). On the equation

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

of degree 6

COMMENTARII MATHEMATICI HELVETICI. vol. 3, pp. 605-617, (2004)

My papers

91 De Concini, C.; Procesi, C., Rogora, E., Zabrocki, On equations of degree 3?2*m*

In ROFMAN. Math, Notae. (vol. 42, pp. 109-122). Boletin del Instituto de Matematica "Beppo Levi". Univ. Nac. Rosario. (2004)

92 De Concini, C.;Procesi, C. Nested sets and Jeffrey Kirwan cycles Geometric methods in algebra and number theory, 139-149, Progr. Math., 235, Birkhauser Boston, Boston, MA, (2005)

93 De Concini, C.;Procesi, C. On the geometry of graph arrangements

In BOGOMOLOV, FEDOR, TSCHINKEL, YURI. Geometric Methods in Algebra and Number Theory. (vol.235). Progress in Mathematics, Band 235. BASEL: Ein Birkhauser Buch (SWITZERLAND).



94 De Concini, C.;Procesi, C., N.Reshetikhin, M.Rosso. Hopf algebras with trace and representations
Invent. Math. 161, no. 1, 1–44 (2005)
95 De Concini, C.;Procesi, C. On the geometry of toric arrangements
Transform. Groups 10, no. 3-4, pp. 387-422 (2005)
96 C. Procesi, Lie Groups: An Approach Through Invariants and Representations (Universitext) Springer 2006



97 De Concini C., C. Procesi A curious identitity and the volume of the root spherical complex,

Rend. Lincei, s. IX, v.XVII, n.2, pp.155-165

98 De Concini C., C. Procesi (2008). The zonotope of a root system.

Transformation Groups, vol. 13; p. 507-526,

99 De Concini C., C. Procesi Topics in hyperplane arrangements, polytopes and box–splines (libro in stampa presso Springer)



- 100 De Concini C., C. Procesi (2008). Hyperplace arrangements and box splines.
- MICHIGAN MATHEMATICAL JOURNAL, vol. 57; p. 201-225,
- 101 C. De Concini, C. Procesi. M. Vergne (2010). Vector partition functions and index of transversally elliptic operators.
- Transformation Groups, vol. 15 n. 4; p. 775-811, ISSN: 1083-4362
- 102 C. De Concini, C. Procesi. M. Vergne (2010). Vector partition function and generalized Dahmen-Micchelli spaces.
- Transformation Groups, vol. 15 n.4; p. 751-773, ISSN: 1083-4362



103 C. De Concini, C. Procesi. M. Vergne, The infinitesimal index, arXiv:1003.3525

104 C. De Concini, C. Procesi. M. Vergne, arXiv:1012.1049 Box splines and the equivariant index theorem

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

105 C. De Concini, C. Procesi. M. Vergne, arXiv:1005.0128 Infinitesimal index: cohomology computations Non com. Algebra Invariants Enumerative geometry Springer's Theory Quantum groups Knots n! Index and splines PDE's



106, M. Procesi, C. Procesi, Normal Form for the Schrödinger equation with analytic non–linearities, preprint arXiv:1012.0446 107, M. Procesi, C. Procesi, A KAM algorithm for the non–linear Schrödinger equation (in preparazione)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●