From maximal stability to minimal stability: a paradigm change

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Abstract

A traditional focus in mechanics has been traditionally on the study of most stable configurations characterized as global minima of the appropriate energy functionals. More recently the emphasis has been shifted to the study of local minima describing metastable states. Among the local minima a subclass of marginally stable states plays a dominant role in the study of non-equilibrium steady regimes of quasi-statically driven dissipative systems. It was realized that the minimally stable states not only delineate the failure thresholds but also serve as attractors of the corresponding dynamical systems. The most striking feature of such marginally stable dynamics is the power law structure of fluctuations signaling a highly correlated behavior at all scales (criticality). An incomplete list of mechanical systems exhibiting this behavior includes friction, earthquakes, plasticity, damage and martensitic phase transitions. Marginal stability has been also realized as the main feature of granular systems near jamming and of a broad range of biological systems from cytoskeleton to brain. While a general mathematical theory of marginally stable evolution is still absent several illuminating examples have been partially understood. In this short course we address a recent progress in the modeling of systems in marginal equilibrium and formulate some open problems.

In Lecture 1 we discuss the simplest zero-dimensional problems taken from the theory of friction and consider in some detail a one dimensional problem from the theory of transformational plasticity in shape memory alloys. In Lecture 2 we consider a two dimensional problem from plasticity theory showing the emergence of critical fluctuations. In Lecture 3 we discuss various sides of near-buckling regimes playing an unusually important role in the behavior of granular media and even in vertebrate morphogenesis. In Lecture 4 we formulate a rigorous definitions of strong and weak material stability and reveal their link to nucleation of defects. If time allows we also discuss an example of sustained marginal stability and criticality in the mechanical response of skeletal muscles.

Some references:

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