Università La Sapienza di Roma - Seminar

3D-Reconstruction: Some Results

Jean-Denis DUROU

IRIT - Université Paul Sabatier - Toulouse

CNRS
INPT
UPS
UT1
Institut de Recherche en Informatique de Toulouse

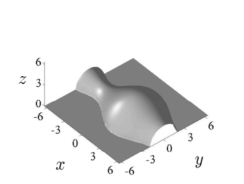
Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

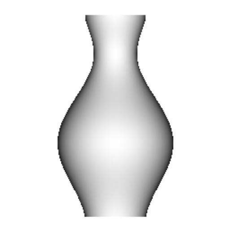
Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

Direct problem or inverse problem?



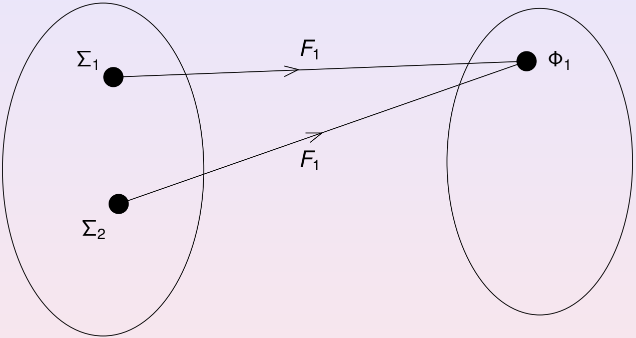
$\Phi = F(\Sigma)$



Scene Σ Camera F Photograph Φ

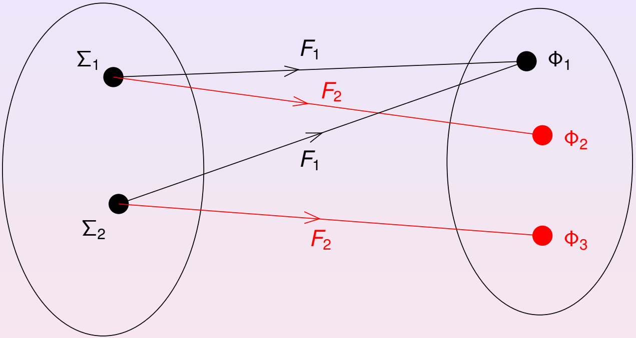
Data	Unknown	Technique	Problem
Σ, F	Φ	Image synthesis	Direct
Φ, F	Σ	3D-reconstruction	Inverse

Well-posed problem or ill-posed problem?



Data	Solutions	Technique	Problem
(Φ_1, F_1)	Σ_1 ou Σ_2 ?	Monoscopic vision	Ill-posed
(Φ_1, F_1) et (Φ_2, F_2)	Σ_1	Stereoscopic vision	Well-posed

Well-posed problem or ill-posed problem?



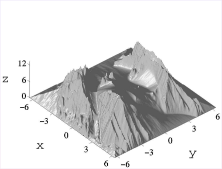
Data	Solutions	Technique	Problem
(Φ_1, F_1)	Σ_1 ou Σ_2 ?	Monoscopic vision	Ill-posed
(Φ_1, F_1) et (Φ_2, F_2)	Σ_1	Stereoscopic vision	Well-posed

Monoscopic vision

Original photograph

Φ_1

? →



Reconstructed shape using SFS

$\Sigma = F^{-1}(\Phi_1)$

F →

Synthetic image

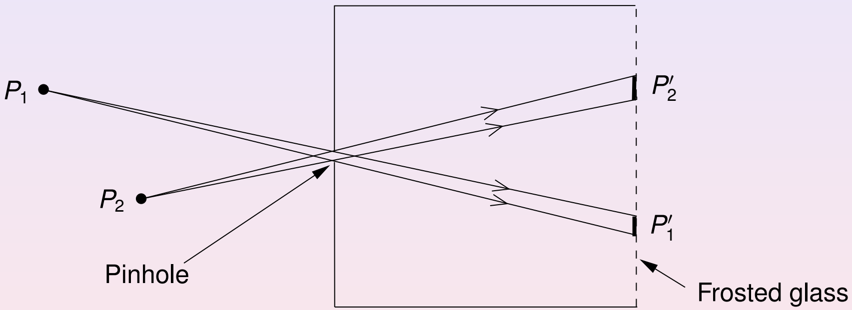
$\Phi_2 = F(\Sigma)$

- Interesting for face recognition?
- An idea for image cryptation!

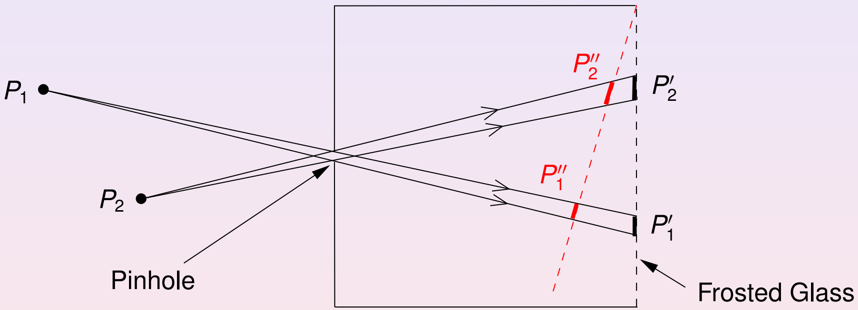
Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

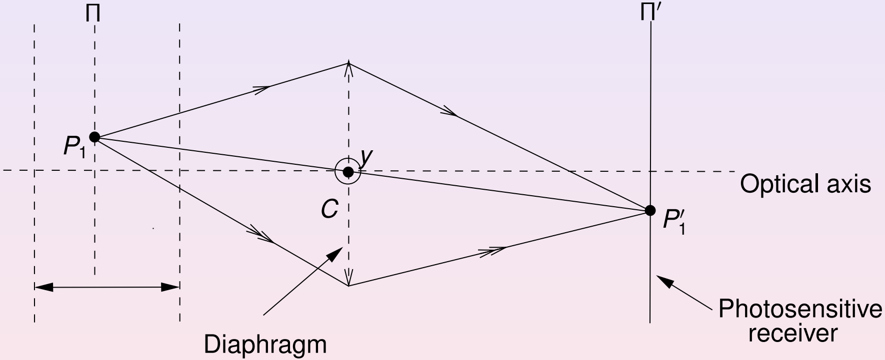
Principle of the darkroom



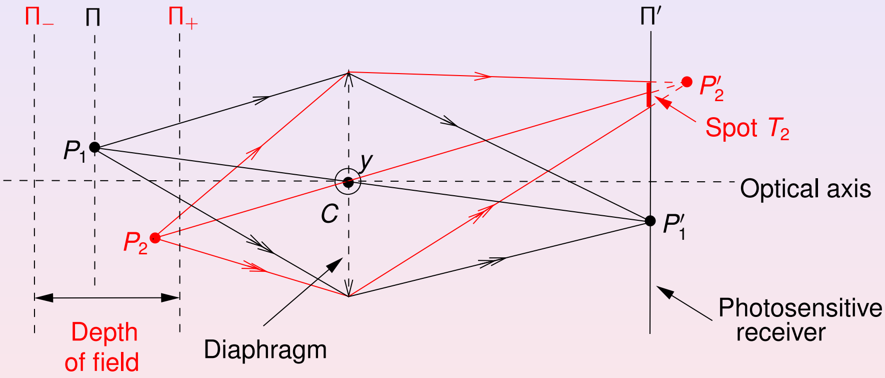
Principle of the darkroom



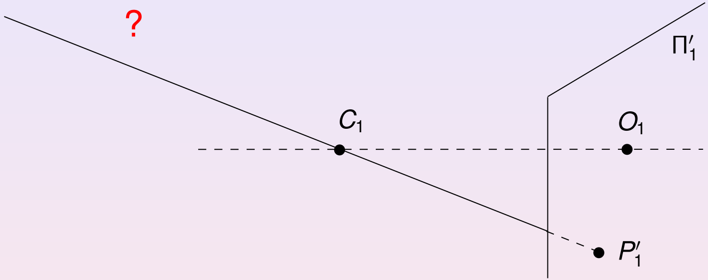
Main interest of photography



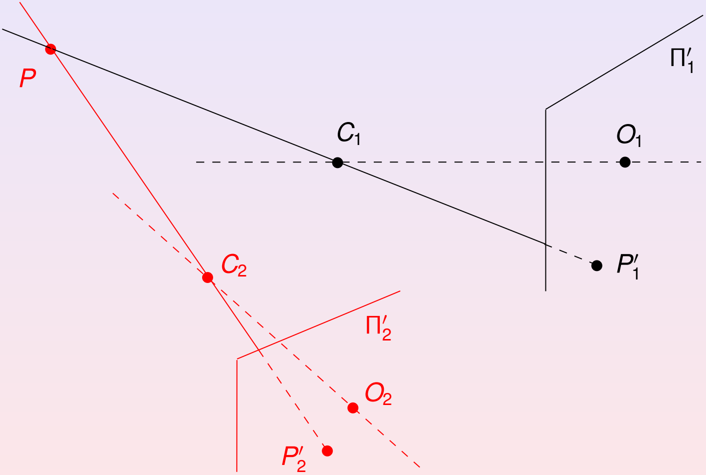
Main interest of photography



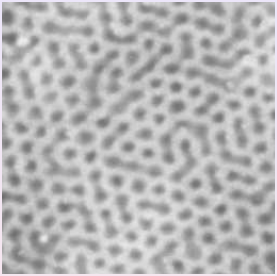
Main weakness of photography



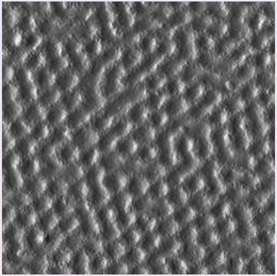
Main weakness of photography



Depth images



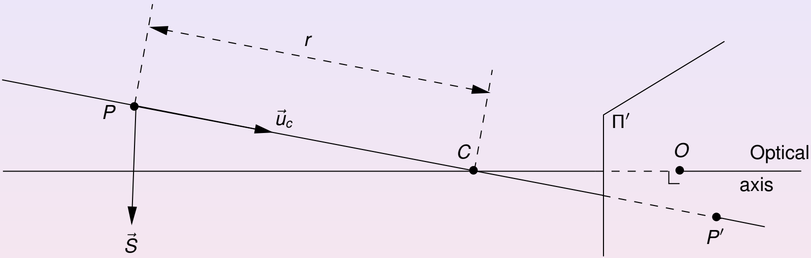
Depth image



Slope image

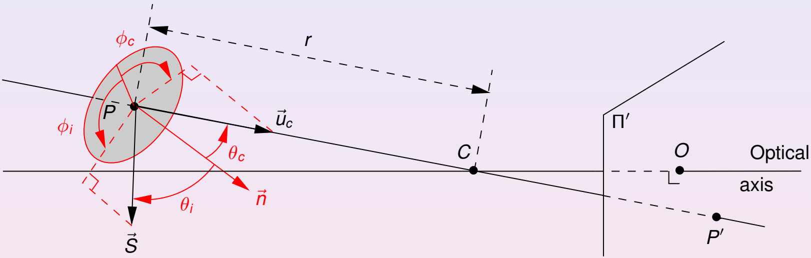
AFM images

Shape-from-shading: an ill-posed problem



- Data: greylevel g in P'
- Unknowns: r, θ_c, ϕ_c, θ_i and ϕ_i
- Five unknowns for one equation: ill-posed problem

Shape-from-shading: an ill-posed problem



- Data: greylevel g in P'
- Unknowns: r, θ_c, ϕ_c, θ_i and ϕ_i
- Five unknowns for one equation: ill-posed problem

The most famous shape-from-shading modelling

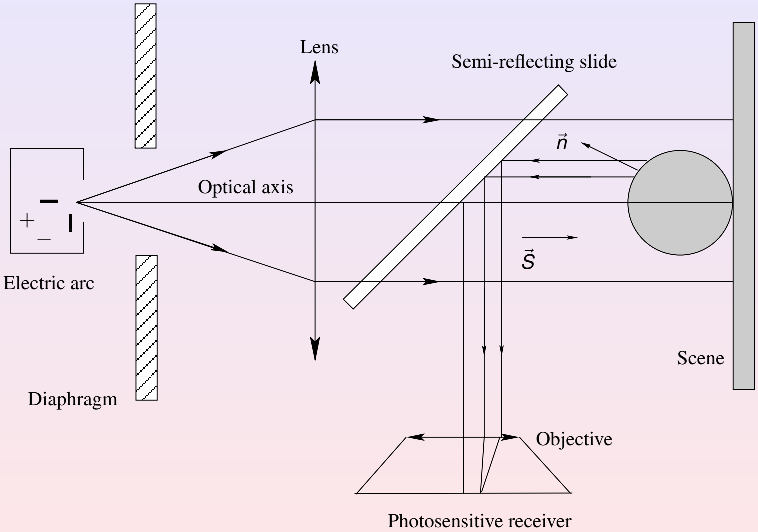
Eikonal equation

- Partial Differential Equation (PDE) of the first order
- Non-linear equation

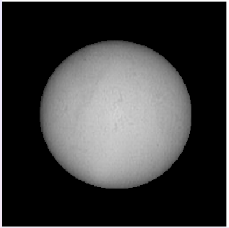
Required assumptions

- The photograph is sharp
- The camera is photometrically calibrated
- The inter-reflections can be neglected
- The surface is Lambertian
- The lighting is parallel, uniform and frontal
- The perspective projection can be neglected
- The scene surface is differentiable

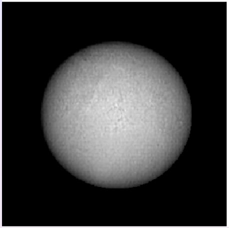
Satisfaction of the required assumptions



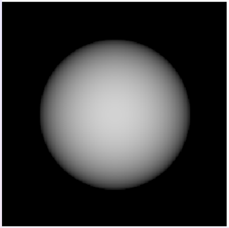
Validation of the obtained photographs



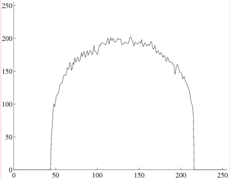
Original photograph



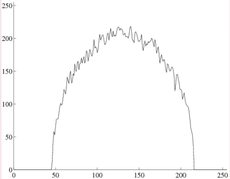
Corrected photograph



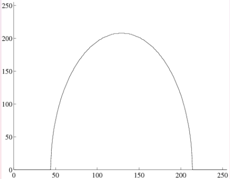
Synthetic image



Cut of the original photograph

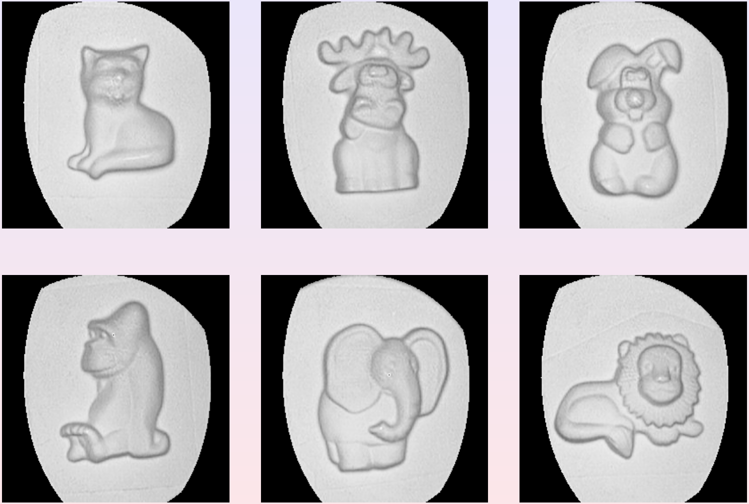


Cut of the corrected photograph



Cut of the synthetic image

Photographs which conform to the eikonal equation



Bibliography on shape-from-shading (www.irit.fr/sfs)

Several communities implied

- 1950 → 1970: astronomy
- 1970 → 1990: computer vision
- 1990 → today: applied mathematics

Great dispersal of the publications

- 1989: book by Horn and Brooks, MIT Press (250 published papers)
- 1999: state of the art by Zhang *et al.*, PAMI (750 published papers)
- 2008: state of the art by Durou, Falcone and Sagona, CVIU (1200 published papers)

Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

Three classes of methods of resolution

Methods of resolution of PDEs (Falcone and Sagona, ICIAP 1997)

- Boundary condition required
- Exact solution Σ: $F(\Sigma) = \Phi$
- Difficult to implement

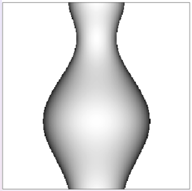
Methods using optimization (Daniel and Durou, ACCV 2000)

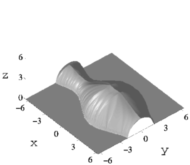
- Boundary condition not required
- Approached solution Σ: $F(\Sigma) \approx \Phi$
- Difficult to implement

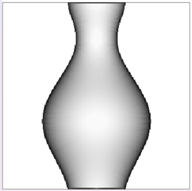
Methods approximating the modelling (Tsai and Shah, CVPR 1992)

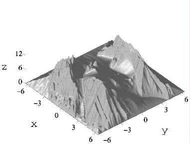
- Boundary condition required
- Approached solution Σ: $F(\Sigma) \approx \Phi$
- Easy to implement

Reconstructions using Falcone and Sagona’s method

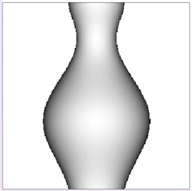


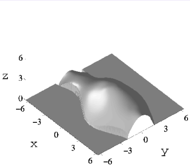


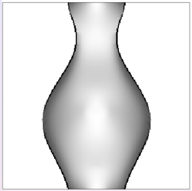


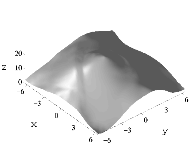


Reconstructions using Daniel and Durou's method

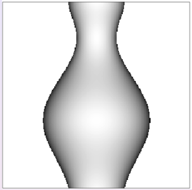


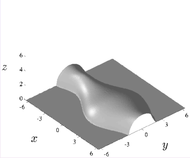


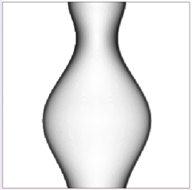


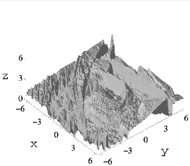


Reconstructions using Tsai and Shah's method





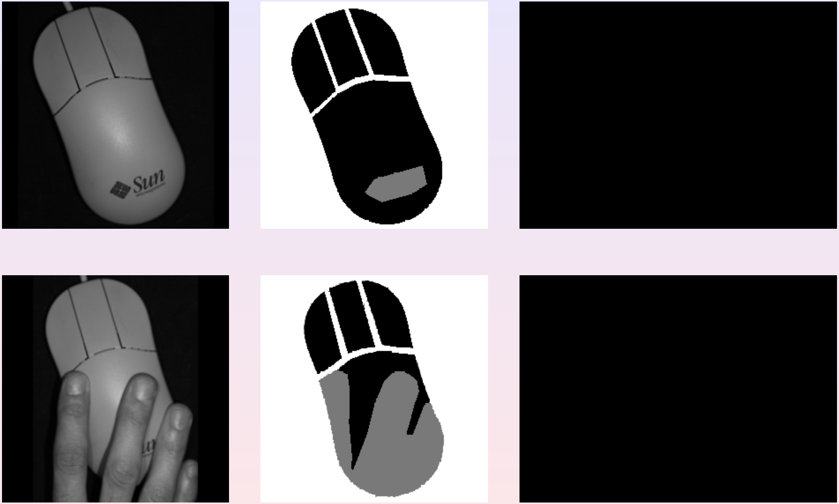




Use of a global surface model

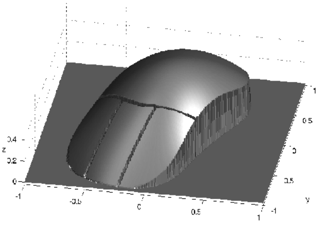
Photographs | “Reliable” pixels | Reconstructed surfaces

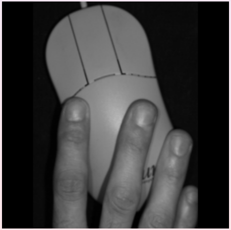
Use of a global surface model

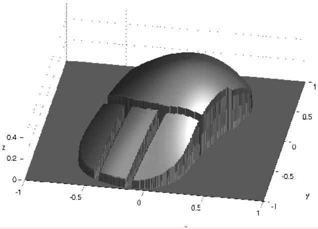


Photographs   “Reliable” pixels   Reconstructed surfaces

Use of a global surface model

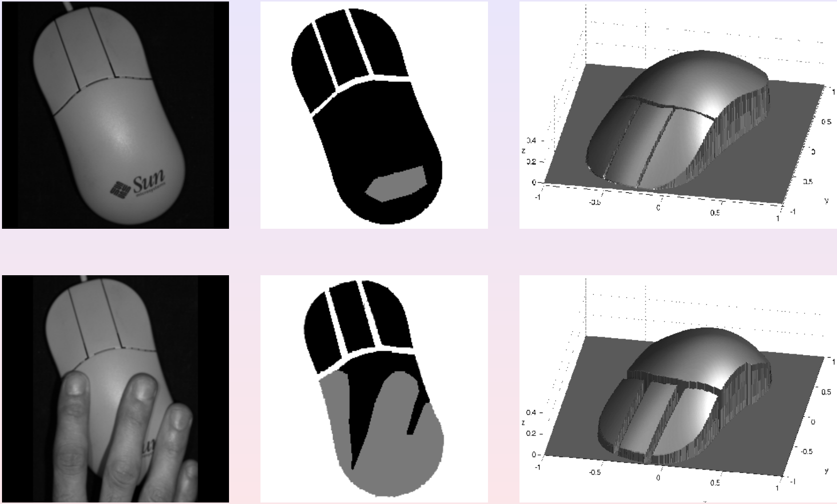






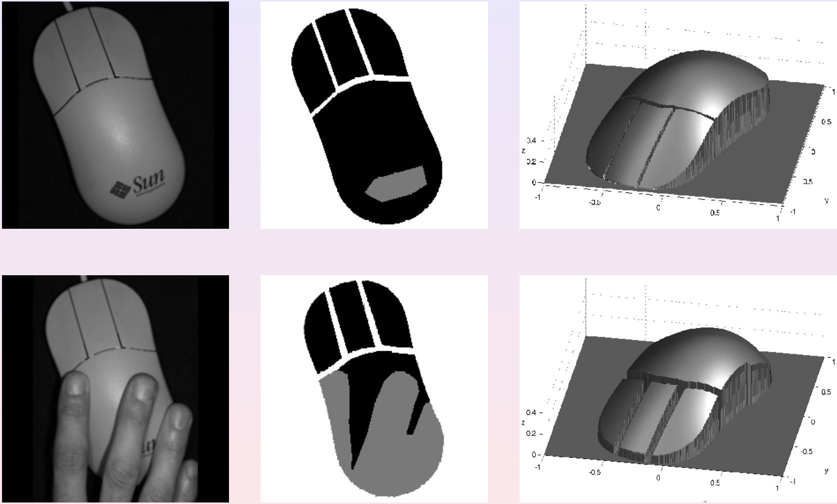
Photographs — “Reliable” pixels — Reconstructed surfaces

Use of a global surface model



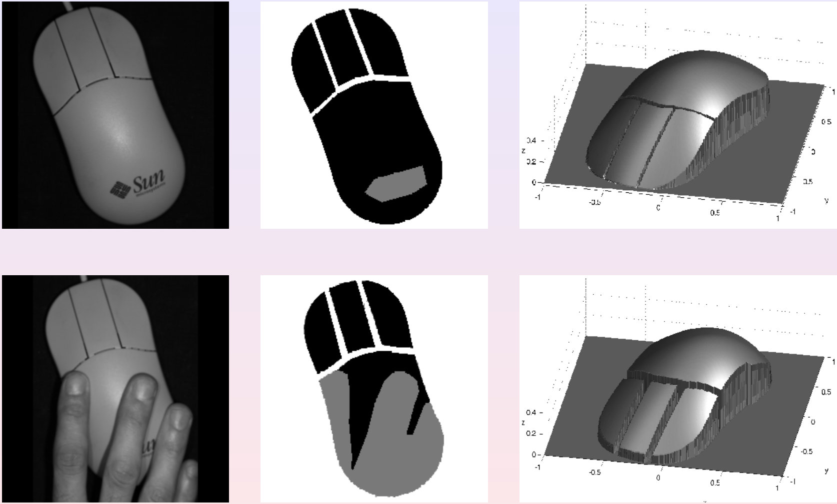
Photographs "Reliable" pixels Reconstructed surfaces

Use of a global surface model



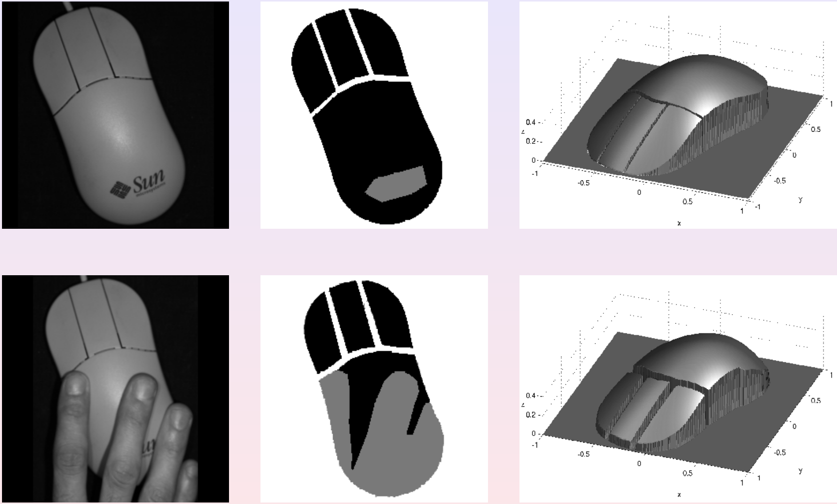
Photographs | “Reliable” pixels | Reconstructed surfaces

Use of a global surface model



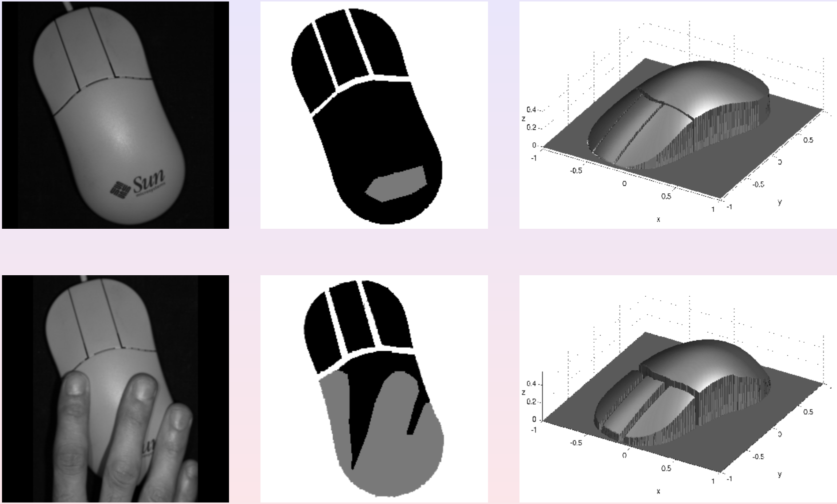
Photographs — “Reliable” pixels — Reconstructed surfaces

Use of a global surface model



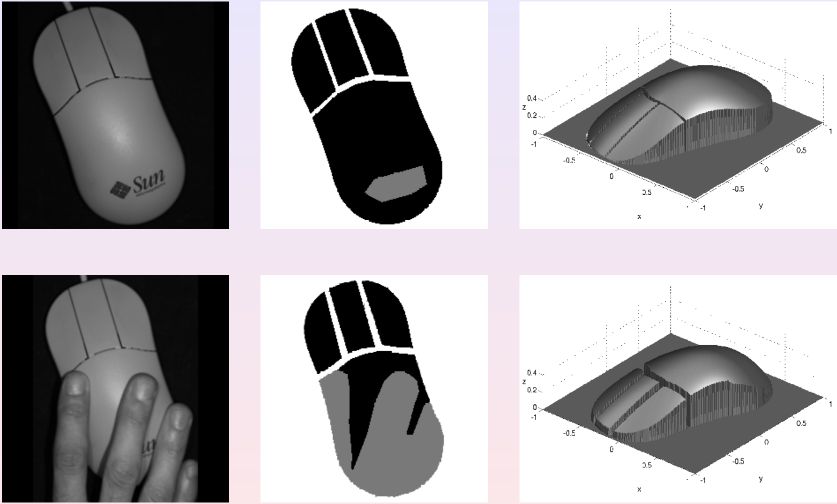
Photographs — “Reliable” pixels — Reconstructed surfaces

Use of a global surface model



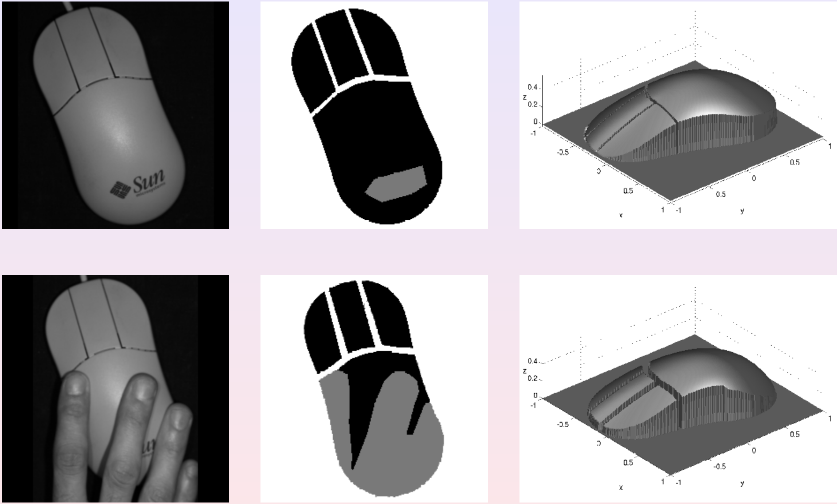
Photographs | “Reliable” pixels | Reconstructed surfaces

Use of a global surface model



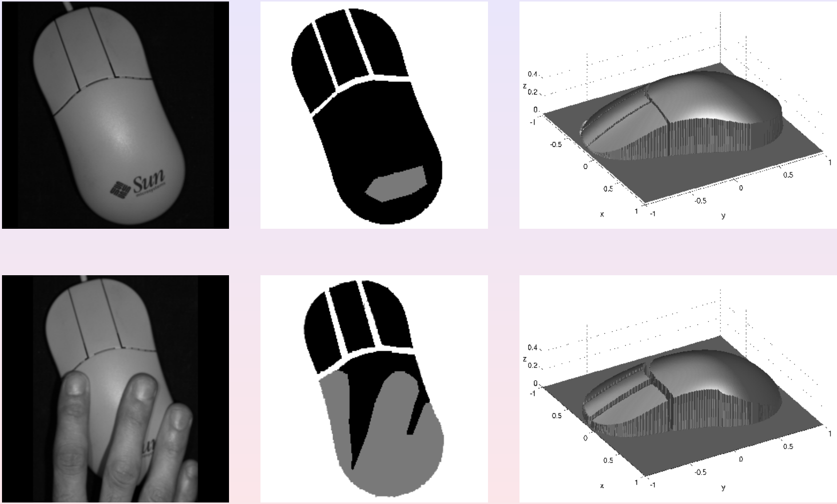
Photographs | “Reliable” pixels | Reconstructed surfaces

Use of a global surface model



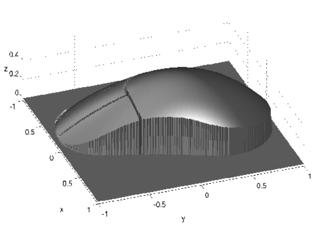
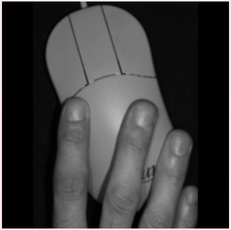
Photographs — "Reliable" pixels — Reconstructed surfaces

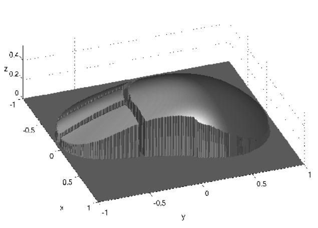
Use of a global surface model



Photographs — “Reliable” pixels — Reconstructed surfaces

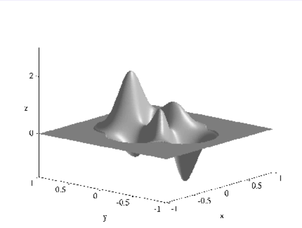
Use of a global surface model



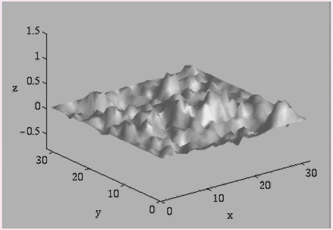
Photographs | "Reliable" pixels | Reconstructed surfaces

Use of stochastic optimization



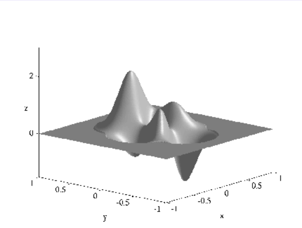
Real shape

Synthetic image



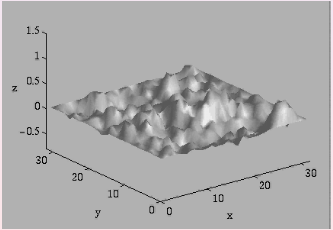
Reconstructed shape

Use of stochastic optimization



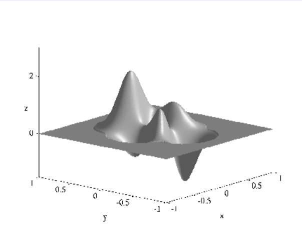
Real shape

Synthetic image



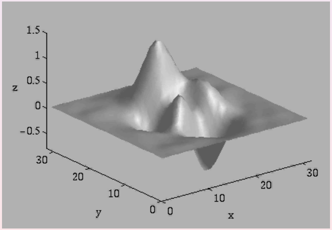
Reconstructed shape

Use of stochastic optimization



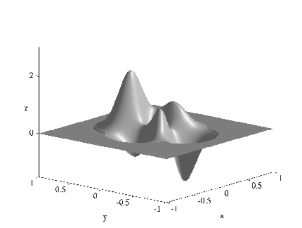
Real shape

Synthetic image



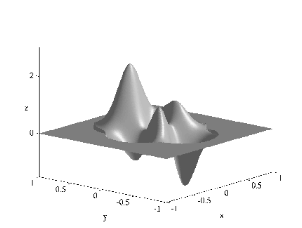
Reconstructed shape

Global surface model and stochastic optimization



Real shape

Synthetic image



Reconstructed shape

- Global surface model: B-spline
- Stochastic optimization: simulated annealing

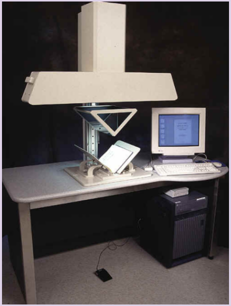
Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

Documents digitization

Various devices

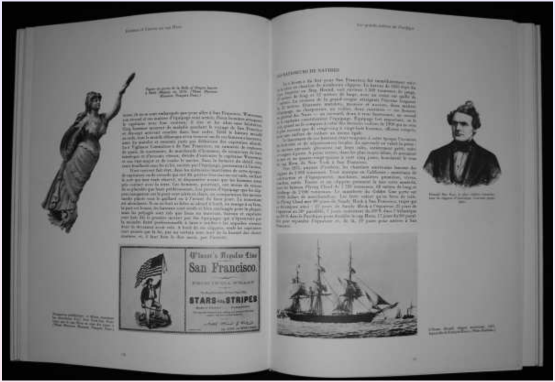
Flatbed scanner



Book scanner

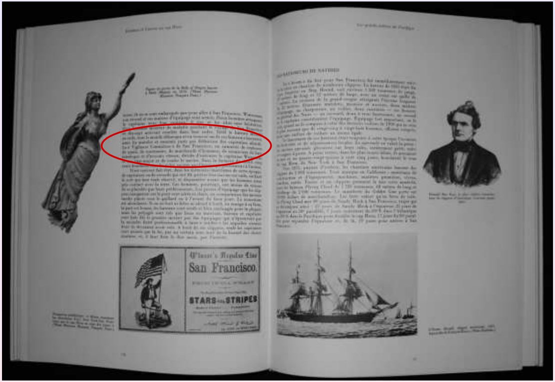
Digital camera

Defaults of books photographs



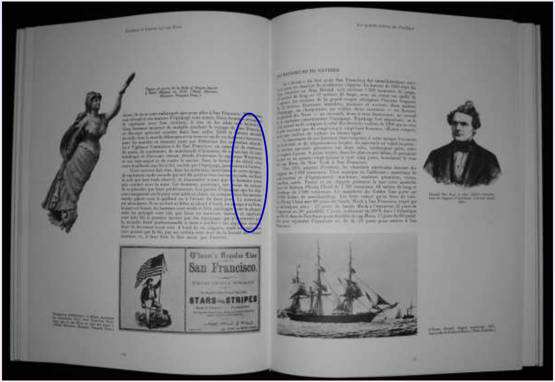
- The lines of text are distorted
- Some characters are shortened
- The greylevel of the non-inked paper is not uniform

Defaults of books photographs



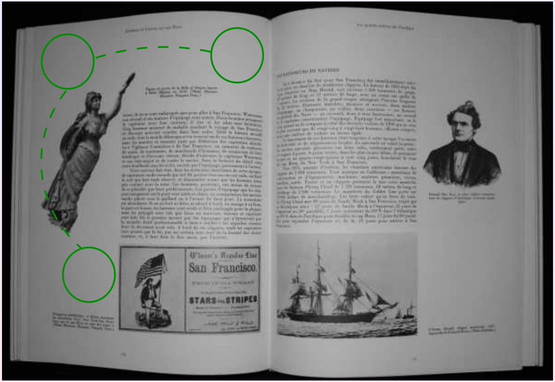
- The lines of text are distorted
- Some characters are shortened
- The greylevel of the non-inked paper is not uniform

Defaults of books photographs



- The lines of text are distorted
- Some characters are shortened
- The greylevel of the non-inked paper is not uniform

Defaults of books photographs



- The lines of text are distorted
- Some characters are shortened
- The greylevel of the non-inked paper is not uniform

Document flattening using shape-from-shading

Example 1

Original photograph

Figure de proue de la *Belle of Oregon* lancée à Bath (Maine), en 1876. (*Photo Mariner Museum, Newport News.*)

ment, ils ne se sont embarqués que pour aller à San Francisco. Waterman, son second et ses maîtres d'équipage sont armés. Deux hommes attaquent le capitaine avec leur couteau; il tire et les abat sans hésitation. Cinq hommes meurent de maladie pendant le voyage de San Francisco et dix-sept arrivent couchés dans leur cadre. Sitôt le bateau mouillé en rade, tout le monde débarque et va trouver un de ces fameux « lawyers », amis du matelot et ennemis jurés par définition des capitaines abusifs. Le « Vigilance Committee » de San Francisco, un ramassis de traîneurs de quais, de souteneurs, de marchands d'hommes, de tenanciers de bars interlopes et d'avocats véreux, décide d'exécuter le capitaine Waterman et son état-major et de couler le navire. Sans la fermeté du shérif, cinq cents braillards sans foi ni loi, excités par l'équipage, passeraient à l'action.

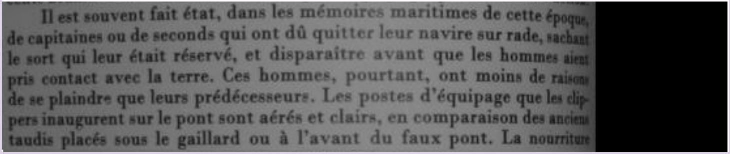
Il est souvent fait état, dans les mémoires maritimes de cette époque, de capitaines ou de seconds qui ont dû quitter leur navire sur rade, sachant le sort qui leur était réservé, et disparaître avant que les hommes aient pris contact avec la terre. Ces hommes, pourtant, ont moins de raisons de se plaindre que leurs prédécesseurs. Les postes d'équipage que les clippers inaugurent sur le pont sont aérés et clairs, en comparaison des anciens taudis placés sous le gaillard ou à l'avant du faux pont. La nourriture est abondante. Si on ne boit ni bière ni alcool à bord, on mange à sa faim, la paie est bonne, les bateaux sont neufs et bien aménagés pour la plupart mais les préjugés sont tels que bons ou mauvais, bateau et capitaine sont haïs dès le premier instant par des équipages qui n'éprouvent pas la moindre fierté professionnelle à faire « rendre » les superbes oiseaux dont ils devraient avoir soin. A bord de ces clippers, seuls les capitaines sont animés par la foi, par un certain sens inné de la beauté des choses marines, et, il faut bien le dire aussi, par l'intérêt.

Winsor's Regular Line
San Francisco.
FROM INDIA WHARF
STARS AND STRIPES

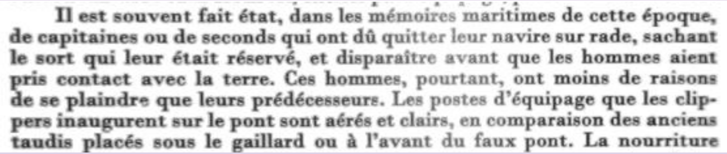
Corrected image

Document flattening using shape-from-shading

Zoom on the area close to the binding


Il est souvent fait état, dans les mémoires maritimes de cette époque, de capitaines ou de seconds qui ont dû quitter leur navire sur rade, sachant le sort qui leur était réservé, et disparaître avant que les hommes aient pris contact avec la terre. Ces hommes, pourtant, ont moins de raisons de se plaindre que leurs prédécesseurs. Les postes d'équipage que les clippers inaugurent sur le pont sont aérés et clairs, en comparaison des anciens taudis placés sous le gaillard ou à l'avant du faux pont. La nourriture

Original photograph


Il est souvent fait état, dans les mémoires maritimes de cette époque, de capitaines ou de seconds qui ont dû quitter leur navire sur rade, sachant le sort qui leur était réservé, et disparaître avant que les hommes aient pris contact avec la terre. Ces hommes, pourtant, ont moins de raisons de se plaindre que leurs prédécesseurs. Les postes d'équipage que les clippers inaugurent sur le pont sont aérés et clairs, en comparaison des anciens taudis placés sous le gaillard ou à l'avant du faux pont. La nourriture

Corrected image

Document flattening using shape-from-shading

Example 2

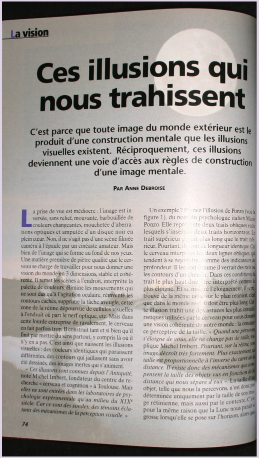
Scanned image

Original photograph

Corrected image

Other method using shape-from-contour

Example 3



Original photograph

Corrected image

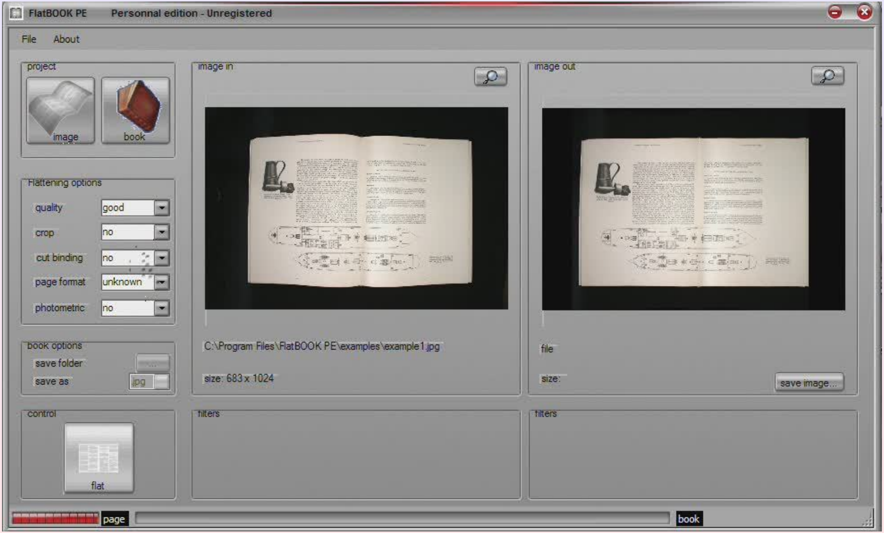
Other method using shape-from-contour

Example 4

Original photograph

Corrected image

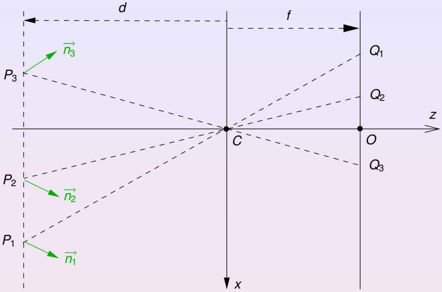
FlatBOOK: A software by Frédéric COURTEILLE



Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

Problematic



- Data: a field of unit normals

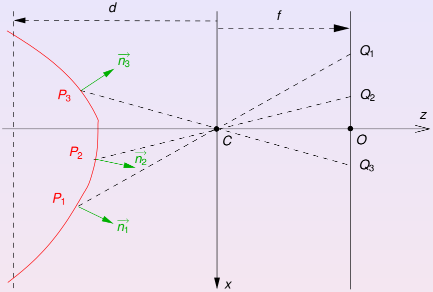
  $\vec{n}(x, y) = (n_1(x, y), n_2(x, y), n_3(x, y))$

  $\|\vec{n}\| = 1 \rightarrow$ 2 degrees of freedom

- Unknown: the scene surface

  $P(x, y) = (X(x, y), Y(x, y), Z(x, y))$

Problematic



- Data: a field of unit normals
  $$\vec{n}(x, y) = (n_1(x, y), n_2(x, y), n_3(x, y))$$
  $$\|\vec{n}\| = 1 \rightarrow 2 \text{ degrees of freedom}$$

- Unknown: the scene surface
  $$P(x, y) = (X(x, y), Y(x, y), Z(x, y))$$

Interest for 3D-reconstruction

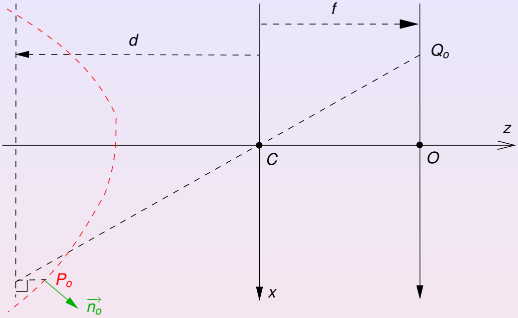
Photometric methods

- *Shape-from-shading*
  Computing the normal field (not necessary): ill-posed problem
- Stereophotometry
  Computing the normal field: well-posed problem
  $\longrightarrow$ Validation of our new method of integration of a normal field

Geometric methods

- *Shape-from-texture*
  Computing the normal field: well-posed problem

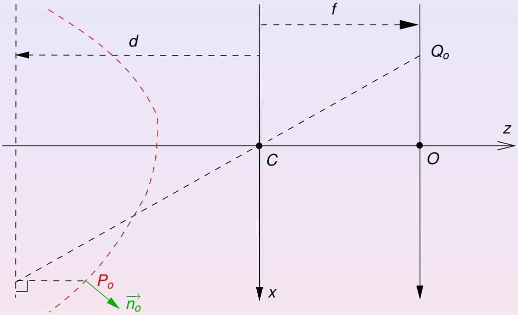
Orthographic projection



- Normal to point Q_o: $\overrightarrow{n_o} = (n_{1,o}, n_{2,o}, n_{3,o})$
- Conjugated objet point: $P_o = (X_o, Y_o, Z_o)$

Fonction Z_o can be computed only up to an additive constant

Orthographic projection



- Normal to point Q_o: $\vec{n_o} = (n_{1,o}, n_{2,o}, n_{3,o})$
- Conjugated objet point: $P_o = (X_o, Y_o, Z_o)$

Fonction Z_o can be computed only up to an additive constant

Orthographic gradient equations

Equations

$$\begin{cases} \dfrac{\partial Z_o}{\partial x} = -\dfrac{d}{f}\,\dfrac{n_{1,o}}{n_{3,o}} \\ \dfrac{\partial Z_o}{\partial y} = -\dfrac{d}{f}\,\dfrac{n_{2,o}}{n_{3,o}} \end{cases}$$

Solution

$$Z_o(Q) = Z_o(Q_{\text{start}}) - \frac{d}{f} \int_{Q_{\text{start}}}^{Q} \frac{n_{1,o}\,dx + n_{2,o}\,dy}{n_{3,o}}$$

Orthographic gradient equations

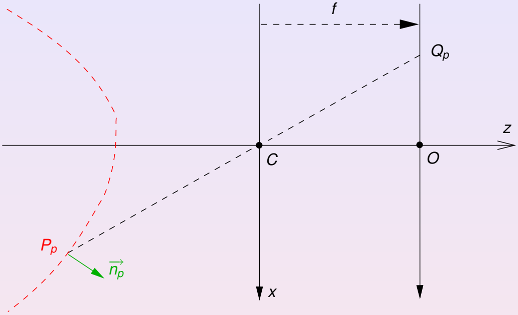
Equations

$$\begin{cases} \dfrac{\partial Z_o}{\partial x} = -\dfrac{d}{f}\,\dfrac{n_{1,o}}{n_{3,o}} \\ \dfrac{\partial Z_o}{\partial y} = -\dfrac{d}{f}\,\dfrac{n_{2,o}}{n_{3,o}} \end{cases}$$

Solution

$$Z_o(Q) = Z_o(Q_{\text{start}}) - \frac{d}{f} \int_{Q_{\text{start}}}^{Q} \frac{n_{1,o}\,dx + n_{2,o}\,dy}{n_{3,o}}$$

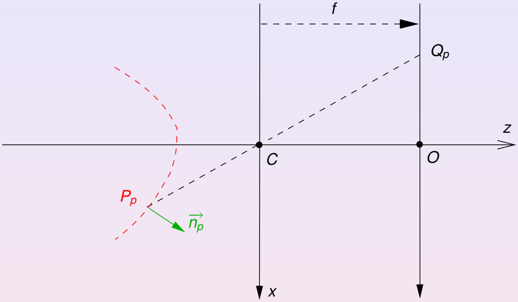
Perspective projection



- Normal to point Q_p: $\overrightarrow{n_p} = (n_{1,p}, n_{2,p}, n_{3,p})$
- Conjugated object point: $P_p = (X_p, Y_p, Z_p)$

Fonction Z_p can be computed only up to a multiplicative constant

Perspective projection



- Normal to point Q_p: $\overrightarrow{n_p} = (n_{1,p}, n_{2,p}, n_{3,p})$
- Conjugated object point: $P_p = (X_p, Y_p, Z_p)$

Fonction Z_p can be computed only up to a multiplicative constant

Perspective gradient equations

Equations

$$\begin{cases} \dfrac{\partial \ln |Z_p|}{\partial x} = -\dfrac{n_{1,p}}{x\, n_{1,p} + y\, n_{2,p} + f\, n_{3,p}} \\ \dfrac{\partial \ln |Z_p|}{\partial y} = -\dfrac{n_{2,p}}{x\, n_{1,p} + y\, n_{2,p} + f\, n_{3,p}} \end{cases}$$

Solution

$$Z_p(Q) = Z_p(Q_{\text{start}})\, e^{-\int_{Q_{\text{start}}}^{Q} \frac{n_{1,p}\, dx + n_{2,p}\, dy}{x\, n_{1,p} + y\, n_{2,p} + f\, n_{3,p}}}$$

Perspective gradient equations

Equations

$$\begin{cases} \dfrac{\partial \ln |Z_p|}{\partial x} = -\dfrac{n_{1,p}}{x\, n_{1,p} + y\, n_{2,p} + f\, n_{3,p}} \\ \dfrac{\partial \ln |Z_p|}{\partial y} = -\dfrac{n_{2,p}}{x\, n_{1,p} + y\, n_{2,p} + f\, n_{3,p}} \end{cases}$$

Solution

$$Z_p(Q) = Z_p(Q_{\text{start}})\, e^{-\int_{Q_{\text{start}}}^{Q} \frac{n_{1,p}\, dx + n_{2,p}\, dy}{x\, n_{1,p} + y\, n_{2,p} + f\, n_{3,p}}}$$

Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

Orthographic projection: two existing methods

Local integration

$$Z_o(Q) = Z_o(Q_{\text{start}}) - \frac{d}{f} \int_{Q_{\text{start}}}^{Q} \frac{n_{1,o}\, dx + n_{2,o}\, dy}{n_{3,o}}$$

Wu and Li's [CVGIP 1988]

Global integration

$$\min_{Z_o} \iint \left[\frac{\partial Z_o}{\partial x} + \frac{d}{f}\,\frac{n_{1,o}}{n_{3,o}}\right]^2 + \left[\frac{\partial Z_o}{\partial y} + \frac{d}{f}\,\frac{n_{2,o}}{n_{3,o}}\right]^2 dx\, dy$$

Horn and Brooks' method [CVGIP 1986]

Orthographic projection: two existing methods

Local integration

$$Z_o(Q) = Z_o(Q_{\text{start}}) - \frac{d}{f} \int_{Q_{\text{start}}}^{Q} \frac{n_{1,o}\,dx + n_{2,o}\,dy}{n_{3,o}}$$

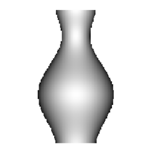
Wu and Li's [CVGIP 1988]

Global integration

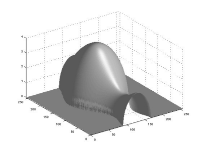
$$\min_{Z_o} \iint \left[\frac{\partial Z_o}{\partial x} + \frac{d}{f}\,\frac{n_{1,o}}{n_{3,o}}\right]^2 + \left[\frac{\partial Z_o}{\partial y} + \frac{d}{f}\,\frac{n_{2,o}}{n_{3,o}}\right]^2 dx\,dy$$

Horn and Brooks' method [CVGIP 1986]

Illustration of these existing methods

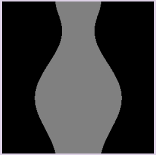


Image

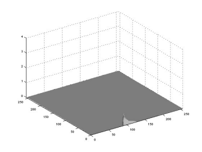


Real shape

Wu et Li's method

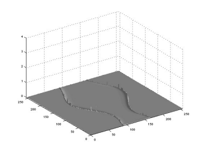


Reconstruction domain



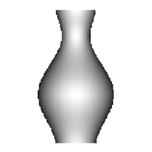
Reconstructed shape

Horn et Brooks' method

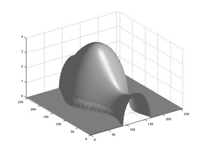


Reconstructed shape

Illustration of these existing methods

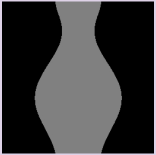


Image

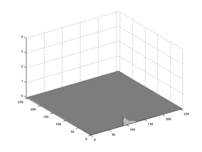


Real shape

Wu et Li's method

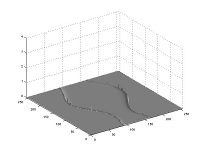


Reconstruction domain



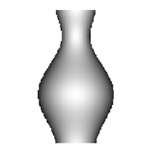
Reconstructed shape

Horn et Brooks' method

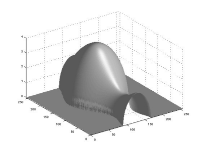


Reconstructed shape

Illustration of these existing methods

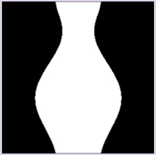


Image

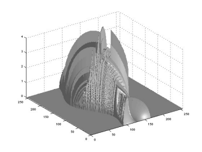


Real shape

Wu et Li's method

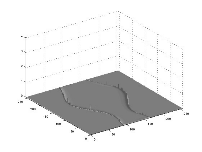


Reconstruction domain



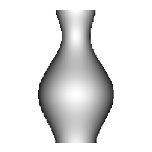
Reconstructed shape

Horn et Brooks' method

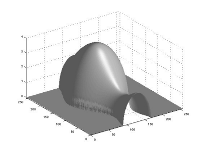


Reconstructed shape

Illustration of these existing methods

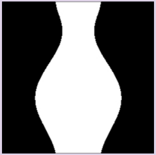


Image

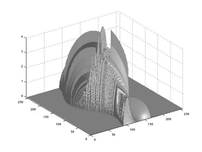


Real shape

Wu et Li's method

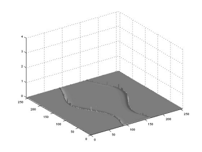


Reconstruction domain



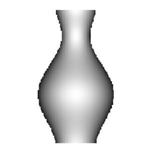
Reconstructed shape

Horn et Brooks' method

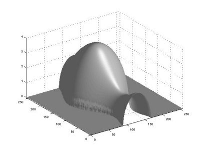


Reconstructed shape

Illustration of these existing methods

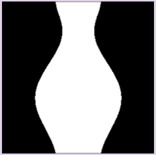


Image

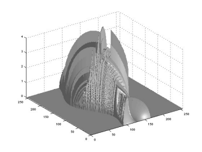


Real shape

Wu et Li's method

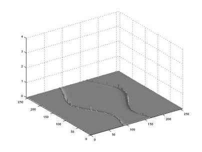


Reconstruction domain



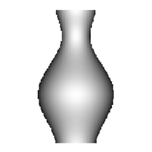
Reconstructed shape

Horn et Brooks' method

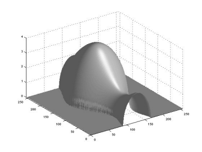


Reconstructed shape

Illustration of these existing methods

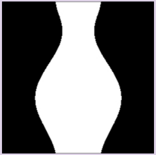


Image

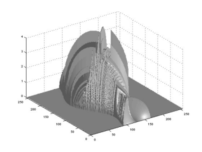


Real shape

Wu et Li's method

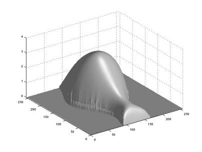


Reconstruction domain



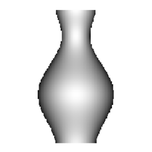
Reconstructed shape

Horn et Brooks' method

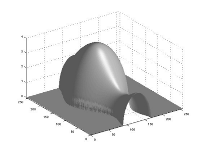


Reconstructed shape

Our new method: no boundary condition is required

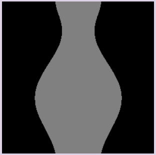


Image

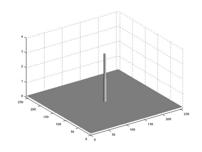


Real shape

Wu et Li's method

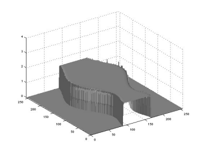


Reconstruction domain



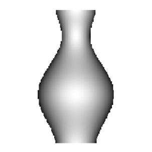
Reconstructed shape

Horn et Brooks' method

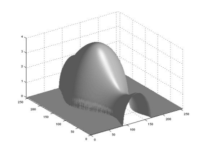


Reconstructed shape

Our new method: no boundary condition is required

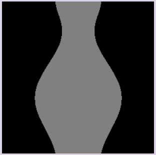


Image

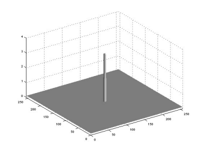


Real shape

Wu et Li's method

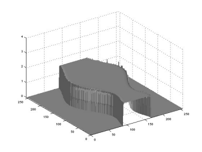


Reconstruction domain



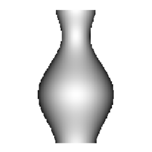
Reconstructed shape

Horn et Brooks' method

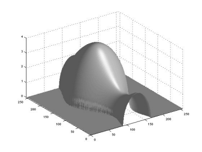


Reconstructed shape

Our new method: no boundary condition is required

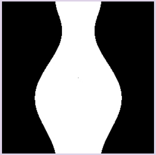


Image

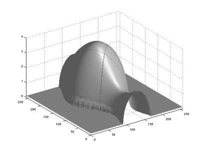


Real shape

Wu et Li's method

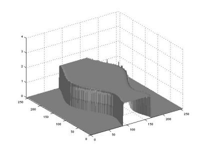


Reconstruction domain



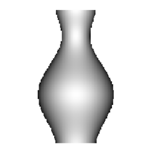
Reconstructed shape

Horn et Brooks' method

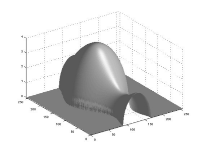


Reconstructed shape

Our new method: no boundary condition is required

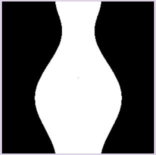


Image

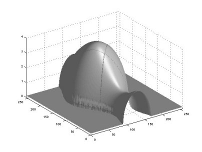


Real shape

Wu et Li's method

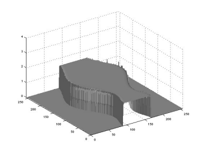


Reconstruction domain



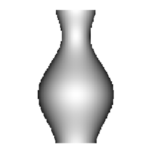
Reconstructed shape

Horn et Brooks' method

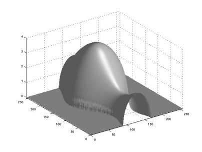


Reconstructed shape

Our new method: no boundary condition is required

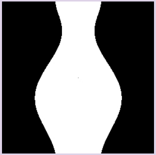


Image

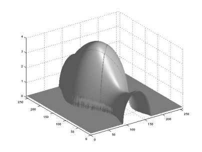


Real shape

Wu et Li's method

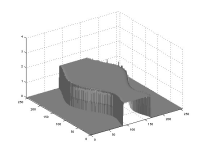


Reconstruction domain



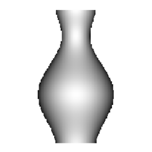
Reconstructed shape

Horn et Brooks' method

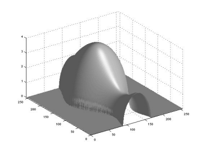


Reconstructed shape

Our new method: no boundary condition is required

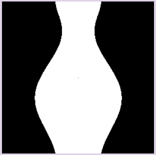


Image

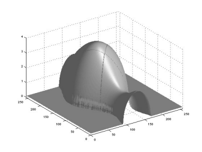


Real shape

Wu et Li's method

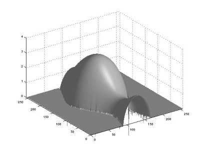


Reconstruction domain



Reconstructed shape

Horn et Brooks' method

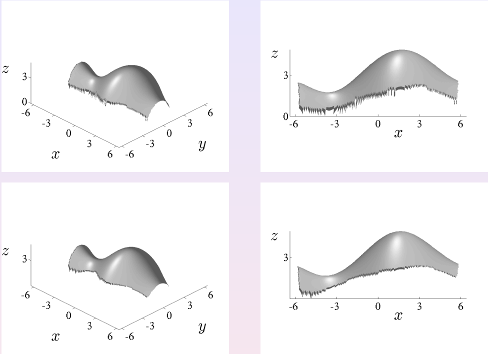


Reconstructed shape

Our new method: taking perspective into account

Real shape

Reconstructed shape



A new method of integration of a normal field

- No boundary condition required
- Taking perspective into account
- Combination of the methods of Wu et Li's and of Horn et Brooks'

Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. **Normal integration: application**
8. Perspectives in 3D-reconstruction

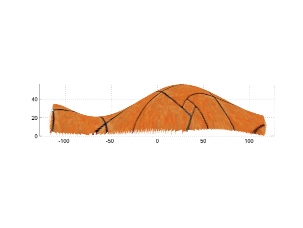
Validation on synthetic images

Assumptions on the images

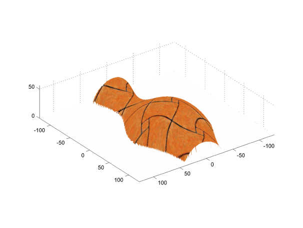
- Non-uniform albedo
- Lambertian surface
- Light at infinity in 5 known directions
- Each visible point is lighted in 3 images at least

Result on synthetic images

Estimate
of the texture



Reconstructed shape
seen from the left



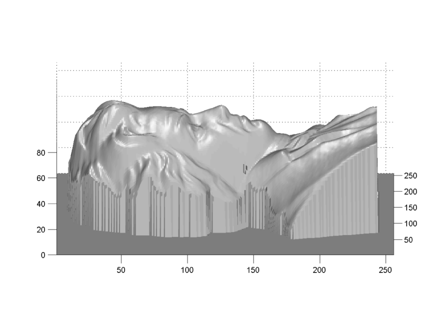
Reconstructed shape
under perspective view

Validation on real images

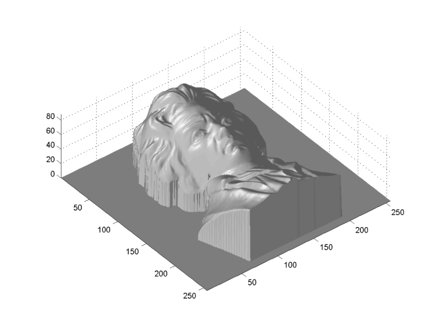
Assumptions on the images (found on the web)

- Uniform albedo
- Lambertian surface
- Light at infinity in 3 known directions
- Each visible point is lighted in 2 images at least

Result on real images



Reconstructed shape
seen from the left



Reconstructed shape
seen under perspective view

Schedule

1. What is 3D-reconstruction?
2. Shape-from-shading: modelling
3. Shape-from-shading: resolution
4. Shape-from-shading: application
5. Normal integration: modelling
6. Normal integration: resolution
7. Normal integration: application
8. Perspectives in 3D-reconstruction

Perspectives on shape-from-shading

Short-term perspectives

- Shape-from-shading and surfaces applicable to planes (*3SD* project)
- Redaction of a monography (continuation of my “Habilitation à Diriger des Recherches”)
- Shape-from-shading and human vision (long stay at the CMLA, ENS Cachan)

Long-term perspectives

- Taking the inter-reflections into account
- Global surface model and stochastic optimization

Other perspectives in 3D-reconstruction

Short-term perspectives

- Stereophotometry: collaboration with Maurizio Falcone?
- Shape-from-shading and shape-from-contour (*3SD* project)

Long-term perspectives

- Combination of shape-from-shading and shape-from-texture
- Use of the photometric methods in a more general framework for 3D-reconstruction?