Numerical methods for optimal control problems: algorithms, analysis and applications

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Book of abstracts

Probabilistic max-plus schemes for solving Hamilton–Jacobi–Bellman equations

Marianne Akian
INRIA and CMAP, École polytechnique CNRS, Palaiseau, France
marianne.akian@inria.fr

We consider fully nonlinear Hamilton-Jacobi-Bellman equations associated to diffusion control problems involving a finite set-valued (or switching) control and possibly a continuum-valued control. We develop lower complexity probabilistic numerical algorithms for such equations by combining max-plus and numerical probabilistic approaches. The max-plus approach is in the spirit of the one of McEneaney, Kaise and Han [3], and is based on the distributivity of monotone operators with respect to suprema. The numerical probabilistic approach is in the spirit of the one proposed by Fahim, Touzi and Warin [2]. A difficulty of the latter algorithm is in the critical constraints imposed on the Hamiltonian to ensure the monotonicity of the scheme, hence the convergence of the algorithm. We shall present new probabilistic schemes which are monotone under rather weak assumptions, including the case of strongly elliptic PDE with bounded derivatives.

This is a joint work with Eric Fodjo, see in particular [1].

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Model order reduction for the control of parametrized PDEs via dynamic programming

Alessandro Alla
Florida State University, Tallahassee, USA
aalla@fsu.edu

The dynamic programming (DP) approach provides a synthesis of optimal feedback controls for many nonlinear optimal control problems. However, once we adopt this approach and compute the value function via the numerical approximation of Hamilton-Jacobi-Bellman (HJB) equation there are two major difficulties: the solutions of an HJB equation are in general non-smooth and the approximation in high dimension requires huge memory allocations.
In this talk, we consider infinite horizon optimal control problems for parametrized partial differential equations. We propose to apply parametric model order reduction techniques to construct low-dimensional subspaces which allows us to approximate the correspondent reduced HJB equation. The subspaces are built upon the algebraic Riccati's equation which provides input-output independent basis functions with information on the value function. Furthermore, to guarantee a low number of basis functions we employ parameter partitioning techniques together with an efficient offline/online splitting of the method. We also present a novel technique to construct a non-uniform grid of the reduced domain based on statistical information. Finally, we discuss numerical examples to illustrate the effectiveness of the proposed method.

Payload optimization for a multi-stage launcher SSO mission using HJB approach

Olivier Bokanowski
Université Paris Diderot, Paris, France
olivier.bokanowski@math.univ-paris-diderot.fr

We consider a payload optimization problem for a three-stage space launcher. The mission is to put a given payload on a sun-synchronous (SSO) orbit. The flight sequence includes two boosts separated by a balistic flight. The first boost steers the launcher to a given transfer orbit. Then, after a ballistic flight, a second boost is used to perform the orbit transfer manoeuver to inject the payload to the targeted SSO orbit. The optimization method presented here is based on the Hamilton-Jacobi-Bellman (HJB) approach for hybrid dynamical systems.
Several issues concerning the application of the HJB approach will be discussed: the HJB framework for hybrid systems, the software solutions (ROC-HJ solver [1]) developped in order to deal with the the curse-of-dimensionality problem for solving a HJB-PDE in high dimensions (dimension is 6 here), the treatment of state-constraints and the trajectory reconstruction procedure adapted to the HJB framework.
This is a joint work with E. Bourgeois, A. Désilles and H. Zidani.

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Optimal control of PDEs with singular arcs

J. Frédéric Bonnans
Inria and CMAP, Ecole Polytechnique, Palaiseau, France
Frederic.Bonnans@inria.fr

Optimal control problems with Hamiltonian depending in an affine way of the control may have singular arcs, i.e. intervals of time where the control is out of bounds (and not determined by other constraints than the bound constraints). Using the Goh transform, one may then obtain some 'strong' second order optimality conditions.
In the field of optimal control of PDEs, this technique has been introduced in [4] in the case of a semilinear heat equation, and generalized in [2, 3] to a general semigroup setting.
We will discuss these results, and more recent work in preparation, extending the state constraint analysis in [1], and the possibility of having several control variables.

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On the solution of some PDE control problems in the framework of the Pontryagin's maximum principle

Alfio Borzì
Universität Würzburg, Würzburg, Germany
alfio.borzi@mathematik.uni-wuerzburg.de

Effective numerical schemes for solving a class of hyperbolic and of parabolic optimal control problems in the framework of the Pontryagin's maximum principle (PMP) are presented.
In the hyperbolic case, the Liouville equation is considered that models the time evolution of a density function and the purpose of the control in the drift is to maximize the measure of a target set at a given final time. In order to solve this problem, a high-order accurate conservative and positive preserving discretization scheme is investigated and a novel iterative optimization method is formulated that solves the PMP optimality condition without requiring differentiability with respect to the control variable.
In the parabolic case, a heat equation with linear distributed control is considered and the purpose of the control is to minimize a discontinuous cost functional of tracking type. Also in this case, a novel iterative scheme is discussed that implements the PMP optimality condition without requiring differentiability with respect to the control variable.
Results of numerical experiments are presented that demonstrate the effectiveness of the proposed solution procedures.
This talk reports on joint works with Souvik Roy [1] and Tim Breitenbach [2].

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Direct numerical solution of cell problems in homogenization of Hamilton-Jacobi equations via generalized Newton's method for inconsistent nonlinear systems

Simone Cacace
Università degli Studi Roma Tre, Roma, Italy
simone.cacace@gmail.com

Homogenization is a well-known and powerful technique for extracting relevant features from complex dynamical systems. In the classical formalism of Hamilton-Jacobi equations, all information is widely compressed into a single nonlinear PDE, the so called *cell problem*, leading to strong non uniqueness phenomena that the Aubry-Mather theory is still trying to catch in very general settings.

From a numerical point of view, these highly ill-posed problems, albeit properly discretized, produce inconsistent systems of nonlinear equations. The selection of a solution requires suitable regularizations, such as the small-δ or the long-time approximations, and also suitable initial guesses or initial data to start the corresponding numerical schemes.

In this talk, we revisit this old fashioned topic in a new perspective, namely we compute numerical solutions to cell problems *directly*, without any artificial regularization, considering the so called *ergodic constants* as they are, additional unknowns. We first discretize the ergodic Hamilton-Jacobi equations using the well-established machinery of numerical schemes for viscosity solutions, then we apply a generalized Newton's method to iteratively move towards a physically relevant solution. Depending on the specific problem under consideration, the resulting linearized Newton's steps translate into inconsistent systems of algebraic equations, for which a unique generalized solution can be provided via the Moore-Penrose pseudoinverse of the corresponding Jacobian, and efficiently implemented by means of suitable QR factorizations. Under very mild differentiability assumptions, possibly treating singularities in a Levenberg-Marquardt fashion, we readily build what is called the *min2norm least-squares* solution for the Newton's step.

Finally, we apply the proposed method to a quite large collection of old and new problems in homogenization of Hamilton-Jacobi equations, aiming to compare our strategy with existing resources in terms of accuracy, convergence and performance, also showing the simplicity of the new method in terms of both parameters tuning and actual code implementation.

To conclude, we present several numerical experiments in dimension one and two, reporting the results obtained by applying our direct method to very well-known problems in the literature of homogenization of Hamilton-Jacobi equations, including the computation of the *effective Hamiltonian* for classical mechanical systems with first order Hamiltonians of eikonal type, or convex with power nonlinearities or nonconvex; weakly coupled first order systems; second order fully nonlinear equations; systems with discontinuous Hamiltoninans and nonlocal velocities, appearing in the homogenization of dislocation dynamics; stationary mean field games in Euclidean spaces or Networks with or without diffusion, single or multi-population; homogenization of mean field games with rapidly oscillating coefficients.

This is a joint work with Fabio Camilli.

The principal eigenvalue for non-variational operators

Italo Capuzzo Dolcetta
Sapienza-Università di Roma, Roma, Italy
capuzzo@mat.uniroma1.it

I will present some recent results concerning a generalized notion of principal eigenvalue for degenerate elliptic operators and discuss its relevance in connection with the validity of the weak maximum principle. This notion applies in particular to non-variational equations arising in ergodic optimal control. This research has been carried on with H. Berestycki, A. Porretta and L. Rossi [1].
I will report also on a finite difference approximation scheme a la Kuo-Trudinger for the computation of the principal eigenvalue based on a min-max formula. This part is due to the collaboration with I. Birindelli and F. Camilli, see [2].

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A multiscale method for reducing the complexity of (controlled) large multi-agent systems

Emiliano Cristiani
IAC–CNR, Rome, Italy
e.cristiani@iac.cnr.it

In this talk we study the possibility of reducing the complexity of a system consisting of a large number of interacting particles. Starting from a system of ordinary/stochastic differential equations describing the process at the microscopic (Lagrangian) level, we obtain its macroscopic (Eulerian) counterpart through a many-particle limit. By suitably coupling the two scales of observation we can reduce the degree of freedom of the microscopic system while maintaining some of its statistical properties. We will describe the multiscale technique in the context of pedestrian [3,4] and opinion dynamics [1,2]. Finally, we discuss the potential of the approach in the framework of controlled differential equations.
Joint work with Andrea Tosin.

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Minimax Differentiability for the Computation of Control, Shape, and Topological Derivatives

Michel C. Delfour
Université de Montréal, Montréal, Canada
delfour@crm.umontreal.ca

A standard approach to the *minimization* of an *objective function* in the presence of *equality constraints* in Mathematical Programming or of a *state equation* in Control Theory is the introduction of Lagrange multipliers or an *adjoint state*, that is, a linear penalization of the equality constraints or the state equation. The initial minimization problem is equivalent to the minimax of the associated Lagrangian. This approach can also be used to compute the one-sided directional derivative with respect to the control or the shape or topology of a family of sets. It is sufficient to consider Lagrangian parametrized by a positive parameter. In this paper we survey some recent results, In particular, by using the new notion of *averaged adjoint* introduced by Sturm [7, 8], the minimax problem need not be related to a saddle point as in Correa-Seeger [1] and the so-called *dual problem* need not make sense. His results have been extended in [6, 5] from the single valued case to the case where the solutions of the state/averaged adjoint state equations are not unique. In such a case, a non-differentiability can occur and only a one-sided directional derivative is expected even if the functions at hand are infinitely differentiable as was illustrated in the seminal paper of Danskin in 1966.
Examples for control and for shape and topological derivatives will be given.

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Recent Progress in Nonlinear Optimal Control Algorithms for Embedded Systems

Moritz Diehl
University of Freiburg, Freiburg, Germany
moritz.diehl@imtek.uni-freiburg.de

When nonlinear optimal control problems are solved on embedded systems for online control and estimation tasks, efficient memory and CPU usage are as important as tailored numerical methods. The basic algorithmic ingredients of all direct optimal control methods are (a) numerical simulation and derivative generation, and (b) the solution of sparse symmetric linear systems or, more general, sparse quadratic programs. In this talk, we review some progress that has been made between 2012 and 2017 in both fields. Most of the algorithms that will be presented and compared in the talk are available as open source (LGPL) code in the C++ optimal control packages ACADO [1], CasADi [2], as well as in the upcoming ANSI-C toolbox acados [3], which all have additional user interfaces to high level environments such as Python, MATLAB, or Octave.

The first new development regards the *Inexact Newton Method with Iterated Sensitivities (INIS)* [4] for stiff ODE and DAE, which tries to combine the main advantage of direct transcription methods– no nested Newton iterations–with a major advantage of shooting methods– the possibility to use inexact matrix factorizations inside the simulation solver. The theoretical highlight of the INIS method is that the local contraction rate of the inexact simulation iteration can be guaranteed also for the optimization iterations under mild conditions.

The second series of new developments regards the efficient solution of sparse quadratic programs as they arise in the direct multiple shooting method. Here, numerical advances in Riccati-based interior point and active set methods were complemented by the development of dense linear algebra routines that are tailored to block sparse embedded optimization problems, which alone turned out to lead to speed-ups of a factor of 2-10, and that have recently be made publicly available in the BLASFEO package [5].

The talk presents joint work with Gianluca Frison, Dimitris Kouzoupis, Andrea Zanelli, Robin Verschueren and Rien Quirynen.

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Fundamental solution semigroups for optimal control

Peter M. Dower
University of Melbourne, Melbourne, Australia
pdower@unimelb.edu.au

Dynamic programming defines a one-parameter semigroup of evolution operators, with respect to a space of terminal payoffs, that describes all possible finite horizon value functions / optimal control problems associated with a specific running payoff. Attendant max-plus (min-plus) linearity and semi-convexity (semi-concavity) properties together provide corresponding convolution representations for these operators, and guarantee existence of one-parameter semigroups of bivariate kernels associated with those representations. Reduced complexity evolution of elements of these kernel semigroups underlies max-plus (min-plus) eigenvector methods (and related sparse approximations) for the computation of value functions for optimal control problems. In this presentation, fundamental solution semigroups are introduced in an optimal control setting, and specific problem classes considered for which the attendant kernel semigroups can be evolved exactly or efficiently, either through exploitation of explicit structure or through approximation. A variety of applications are summarised, including (for example) worst-case analysis for nonlinear systems [1,2], state constrained regulation for linear systems [3], and solving two-point boundary value problems [4,5,6].

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Fractional PDEs Constrained Optimization: An optimize–then–discretize approach with L–BFGS and Approximate Inverse Preconditioning

Fabio Durastante
Università degli Studi dell'Insubria, Como, Italy
fdurastante@uninsubria.it

In this talk, based on [1], we consider the numerical solution of the problem:

$$\begin{cases} \min J(y,u) = \frac{1}{2}\|y - z_d\|_2^2 + \frac{\lambda}{2}\|u\|_2^2, \\ \text{subject to } e(y,u) = 0. \end{cases}$$

where J and e are two continuously Fréchet derivable functionals such that, $J : Y \times U \to \mathbb{R}$, $e : Y \times U \to W$, with Y, U and W reflexive Banach spaces, $z_d \in U$ is given and $\lambda \in \mathbb{R}$ is a fixed positive regularization parameter. The constraint, namely $e(y,u) = 0$, is chosen not to be an ordinary elliptic PDE as in the classic case, but a Fraction Partial Differential Equation: either the *Fractional Advection Dispersion Equation* or the two–dimensional *Riesz Space Fractional Diffusion equation.*
Indeed, many problems that exhibit *non–local properties* have been modeled using fractional calculus, e.g., anomalous diffusion (i.e diffusion not accurately modeled by the usual advection–dispersion equation), the dynamics of viscoelastic and polymeric materials and many others; see, e.g., [2]. We focus on extending the existing strategies for classic PDE constrained optimization to the fractional case. We will present both a theoretical and experimental analysis of the problem in an algorithmic framework based on the L–BFGS method coupled with a Krylov subspace solver. A suitable preconditioning strategy by *approximate inverses* is taken into account as in [3]. Numerical experiments are performed with benchmarked software/libraries thus enforcing the reproducibility of the results.
Joint work with *S. Cipolla* (Università di Roma Tor Vergata).

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Receding-horizon optimal control with economic objectives – practical and asymptotic convergence

Timm Faulwasser
Karlsruhe Institute of Technology, Karlsruhe, Germany
timm.faulwasser@kit.edu

During the last decades, receding-horizon optimal control, which is also termed Model Predictive Control (MPC), has been subject to substantial research efforts from applied mathematicians and control engineers. Recently, there has been a shift in MPC from objective functionals penalizing the deviation from a desired target steady-state to more general performance criteria. MPC schemes using more general objectives are commonly labeled as *economic MPC* [1]. In this context, it has been observed in discrete-time and continuous-time settings that dissipativity and turnpike properties of Optimal Control Problems (OCP) are closely related [2, 3]. Moreover, these properties enable showing that EMPC finds and stabilizes the optimal steady state, without any need for explicit prior knowledge of this steady state. However, it should be noted that without any terminal constraint or terminal penalty in the OCP, one typically shows convergence to a neighborhood of the best steady state, i.e. one establishes *practical* instead of *asymptotic* stability [4, 5].

In this talk, we focus on closing the gap between practical and asymptotic stability in continuous-time EMPC without terminal constraints. To this end, we discuss a notion of exactness of turnpikes in OCPs leading to finite-time convergence of EMPC schemes to the optimal steady state [6]. Furthermore, we present quite general regularity conditions—implying non-exactness of the turnpike in the underlying OCP—under which EMPC, with finite horizon and without terminal constraints or penalties, fails to stabilize the optimal steady state. We investigate the cause of this lack of asymptotic convergence. Moreover, we demonstrate that adding a suitably designed linear end penalty to the OCP leads to asymptotic convergence [7].

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A hybrid control approach to the route planning problem for sailing boats

Adriano Festa
LMI, INSA, Rouen, France
adriano.festa@insa-rouen.fr

We discuss an optimal hybrid control approach [1, 2] to the problem of stochastic route planning for sailing boats, especially in short course fleet races, in which minimum average time is an effective performance index. We show that the hybrid setting is a natural way of taking into account tacking/gybing maneuvers and other discrete control actions, and provide examples of increasing complexity to model the problem. Some tests providing a numerical validation of the approach are shown in good agreement with theoretical and practical knowledge.

Join work with Roberto Ferretti (Roma Tre University) ferretti@mat.uniroma3.it.

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On Model Predictive Control for the Fokker-Planck Equation

Arthur Fleig
Universität Bayreuth, Bayreuth, Germany
arthur.fleig@uni-bayreuth.de

We consider optimal control problems subject to the Fokker-Planck (FP) equation, a second order parabolic partial differential equation. Our motivation to study this problem stems from viewing stochastic processes from a statistical perspective. In this case, rather than controlling the stochastic process in an optimal way, the objective is to steer its underlying probability density function, whose evolution can often be described by the FP equation associated to the process. In this way, the problem is rendered deterministic. A Model Predictive Control (MPC) scheme is then applied to track the solution of this equation over a fixed time horizon.
In this talk, we take a closer look at the stability of the MPC closed loop feedback system. Since we want to avoid stabilizing terminal costs or constraints, the only parameter left to tune in order to guarantee stability is the MPC horizon length. A useful property to this end is the exponential controllability assumption. We apply a technique based on this condition to analyze qualitative changes in the horizon length for various parameters in the problem setting. Of particular interest is the controlled Ornstein-Uhlenbeck process, where the control is time-dependent and either constant or linear in space.
Joint work with L. Grüne (Universität Bayreuth).

Noncommutative aspects of dynamic programming

Stéphane Gaubert
INRIA and CMAP, Ecole polyechnique, CNRS, Université Paris-Saclay
Stephane.Gaubert@inria.fr

McEneaney's max-plus basis method allows one to approximate the value function of a deterministic optimal control problem by a supremum of elementary functions like quadratic forms [5]. Recently, Ahmadi et al. [1] developed an approximation method for Barabanov norms of switched linear systems, relying also on the approximation by suprema of quadratic forms. Related methods allow one to compute program invariants, represented as intersections or unions or ellipsoids. In all these approaches, the solution of large scale linear matrix inequalities by semidefinite programming methods is the computational bottleneck. We will show that the recourse to semidefinite programming can be avoided by expressing invariant generation and value function approximation as fixed point problems in the space of positive semidefinite matrices. These problems involve operators which may be thought of as the noncommutative analogues of dynamic programming operators. These analogues obtained by "tropicalizing" the Kraus maps (completely positive trace preserving maps) which arise in quantum information, i.e., by considering mutivalued suprema of positive semidefinite matrices rather than their sum. This approach relies on several properties of the Löwner order and of non-linear Perron-Frobenius theory, of independent interest, which we will review: selection of minimal upper bounds, contraction of non-linear maps and non-linear flows with respect to several metrics on the cone of positive definite matrices (Thompson, Hilbert and Riemannian metric). This is based on current work with Allamigeon, Goubault, Putot, and Stott [2, 4] and on an earlier work with Qu [3].

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Approaches for bilevel optimal control problems

Matthias Gerdts
Universität der Bundeswehr München, Neubiberg, Germany
matthias.gerdts@unibw.de

The talk addresses bilevel optimal control problems, where the lower level problem is an optimal control problem that depends on parameters of the upper level problem. The upper level problem can be a finite or an infinite dimensional optimization problem. Bilevel optimization problems turn out to be very challenging with regard to both, the investigation of theoretical properties and numerical methods, compare [1]. Typical solution approaches aim at transforming the bilevel structure into a single stage optimization problem. These principal techniques will be summarized briefly.

After that, we discuss two problem classes. In the first, the upper level problem is again an optimal control problem, where the coupling to the lower level problem occurs only through boundary conditions. For this class of problems necessary conditions are derived and applied to an example with a Stackelberg game. To this end, the value function of the lower level problem is exploited in a single level reformulation, compare [2-4]. For more complex problems, numerical methods could be used to approximate the value function, provided the state dimension is low enough. Direct discretization methods are then used to solve the single level optimal control problem together with some smoothing of the value function.

In the second problem class, the upper level problem is a scheduling problem, which aims to find optimal starting times and sequences for the lower level processes in order to, e.g., minimize the total process time. This in fact leads to a mixed-integer bilevel problem. We approach the problem numerically in two ways. The first approach treats the lower level optimal control problems basically as a black box and returns the duration times as functions of the initial times by application of direct solution methods for optimal control problems. Local differentiability of this map can be checked by a parametric sensitivity analysis. However, one cannot expect differentiability or even continuity properties to hold globally for this map. Nevertheless, the numerical approach yields satisfactory results.

The second approach uses an MPEC formulation of the bilevel problem. To this end we exploit the local minimum principle of the continuous optimal control problem. Numerical results from robotics will be presented.

This is a joint work with Konstantin Palagachev.

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Matrix stabilization using differential equations

Nicola Guglielmi
Università dell'Aquila, L'Aquila, Italy
guglielm@univaq.it

We consider the problem of stabilizing a matrix by a correction of minimal norm: Given a square matrix that has some eigenvalues with positive real part, find the nearest matrix having no eigenvalue with positive real part. It can be further required that the correction has a prescribed structure, e.g., to be real, to have a prescribed sparsity pattern, or to have a given maximal rank. We propose and study a novel approach to this non-convex and non-smooth optimization problem, based on the solution of low-rank matrix differential equations. This enables us to compute locally optimal solutions in a fast way, also for higher-dimensional problems. Illustrative numerical experiments provide evidence of the efficiency of the method. It is further shown that the approach applies equally to the related problems of closed-loop stabilization of control systems and to the stabilization of gyroscopic systems.
This is a joint work with Christian Lubich, Universität Tübingen, Germany.

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Performance Verification and Optimal Synthesis of Embedded Optimization-Based Controllers

Colin Jones
EPFL, Lausanne, Switzerland
colin.jones@epfl.ch

The field of fast-MPC, or the use of embedded optimization for high speed control, is a rapidly growing field in academia and increasingly in industry. Achieving the required extremely high speed optimization, often within micro-seconds, on low-end embedded platforms calls for a wide range of heuristic procedures for both the control design, as well as in the implementation of the optimization algorithms themselves. This semi-heuristic process leads to complex control laws that can be very effective, but that are also extremely difficult to tune and design.

This talk will introduce a framework for the non-conservative analysis of many of the heuristics used in these controllers via a convex sum-of-squares approach. We will then build on this framework to develop a formal optimal synthesis procedure for very high-speed embedded optimization-based control laws.

The work presented in this talk was done in collaboration with Ivan Pejcic and Milan Korda and is detailed in the papers listed below.

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Mean field control hierarchy

Dante Kalise
RICAM, Austrian Academy of Sciences, Linz, Austria
dante.kalise@oeaw.ac.at

We model the role of external interventions over a large population as a mean field optimal control problem. Such control problems are constrainted by a PDE of continuity-type, governing the dynamics of the probability distribution of the agent population. We show the existence of mean field optimal controls both in the stochastic and deterministic setting. In this talk, we present a novel approximating hierarchy of sub-optimal controls based on a Boltzmann approach, whose computation requires a very moderate numerical complexity with respect to the one of the optimal control. We provide numerical experiments for models in opinion formation comparing the behavior of the control hierarchy.
Joint work with G. Albi (Università di Verona), M. Fornasier (TU Munich), and Y. P. Choi (Inha University, Seoul).

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Some Examples of Computational Methods for Optimal Control and HJB Equations

Wei Kang
Naval Postgraduate School, Monterey, California, USA
wkang@nps.edu

Two examples are presented using causality free algorithms, in which the value of the solution at a given point is computed without using the value of the solution at any nearby points in the state space. The first example is a 6D HJB equation for the optimal control of rigid bodies. In this example, the causality free algorithm is applied on sparse grids, which have significantly reduced sizes relative to the corresponding dense grids. The computation at grid points is perfectly parallel. The second example is the optimal control of small tailless foam UAVs. Maneuvers of quick speed reduction are simulated. The associated HJB equation has a 4D state space. Once again, sparse grids are used to mitigate the curse of dimensionality and the computational algorithm is causality free.
This work is co-authored with Lucas Wilcox and the work on UAV optimal control is also co-authored with Oleg Yakimenko, both from Naval Postgraduate School.

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Minimum Energy Estimation and Moving Horizon Estimation

Arthur J Krener
Naval Postgraduate School, Monterey, CA, USA
ajkrener@nps.edu

Minimum Energy Estimation is a way of filtering the state of a nonlinear system from partial and inexact measurements. It is a generalization of Gauss' method of least squares. Its application to filtering of control systems goes back at least to Mortenson who called it Maximum Likelyhood Estimation [1]. For linear, Gaussian systems it reduces to maximum likelihood estimation (aka Kalman Filtering) but this is not true for nonlinear systems. We prefer the name Minimum Energy Estimation (MEE) that was introduced by Hijab [2]. Both Mortenson and Hijab dealt with systems in continuous time, we extend their methods to discrete time systems and show how power series techniques can lessen the computational burden.

Moving Horizon Estimation (MHE) is a moving window version of MEE. It computes the solution to an optimal control problem over a past moving window that is constrained by the actual observations on the window. The optimal state trajectory at the end of the window is the MEE estimate at this time. The cost in the optimal control problem is usually taken to be an L2 norm of the three slack variables; the initial condition noise, the driving noise and the measurement noise. MHE requires the buffering of the measurements over the past window. The optimal control problem is solved in real time by a nonlinear program solver but it becomes more difficult as the length of the window is increased.

The power series approach to MME can be applied to MHE and this permits the choice of a very short past window consisting of one time step. This speeds up MHE and allows its real time implementation on faster processes.

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Taylor expansions for the HJB equation associated with a bilinear control problem

Karl Kunisch
University of Graz and Radon Institute, Graz, Austria
karl.kunisch@uni-graz.at

Approximations of the value function associated with a bilinear optimal control problem subject to an infinite dimensional state equation by multilinear forms are derived. The structure of these multilinear forms is suggested by repeated formal differentiation of the associated Hamilton-Jacobi-Bellman equation. The multilinear forms can then be obtained as the solutions to generalized Lyapunov equations with recursively defined right hand sides. They form the basis for defining a suboptimal feedback law. The approximation properties of this feedback law are investigated and an application to the optimal control of a Fokker-Planck equation are given. Numerical examples illustrate the results.
This is a joint work with T. Breiten and L. Pfeiffer.

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Diffusion-Representation Based Methods for Schrödinger Initial Value Problems

William M. McEneaney
University of California San Diego, La Jolla, CA, USA
wmceneaney@ucsd.edu

Diffusion representations have long been a useful tool for solution of second-order Hamilton-Jacobi partial differential equations (HJ PDEs). The bulk of such results apply to real-valued HJ PDEs, that is, to HJ PDEs where the coefficients and solutions are real-valued. The Schrödinger equation is complex-valued, although generally defined over a real-valued space domain, which presents difficulties for the application of solution techniques based on stochastic control representations. Here, a Feynman-Kac approach will be taken to the dequantized form of the Schrödinger equation. However, the representation employs stationarity of the payoff rather than optimization, where the use of stationarity allows one to overcome the limited-duration constraints inherent in methods that use optimization.

Functions of the moments of a set of complex-valued diffusions will be used to obtain an approximation to the solution. If the solution is holomorphic in space, then the approximations converge to the solution as the number of terms approaches infinity. We will specifically consider an example application corresponding to a classical particle in circular motion at a large distance from the origin of a symmetric field.

POD-Based Model Predictive Control with control and state constraints

Luca Mechelli
University of Konstanz, Konstanz, Germany
luca.mechelli@uni-konstanz.de

In the setting of energy efficient building operation, we investigate an optimal boundary control problem governed by linear parabolic convection-diffusion equations with bilateral inequality constraints for the control and the state variables. The aim is to keep the temperature in a room in a certain range with the less possible cost of heating for the heaters for a large time horizon. This leads to model predictive control (MPC) techniques in order to compute the infinite time quadratic cost functional, find the optimal boundary control and approximate the asymptotic behavior of the solution. For the state constraints, in order to gain regular Lagrange multipliers, we utilize a Lavrentiev regularization. After a spatial discretization with finite elements and a time discretization with the implicit Euler method, we solve the problem with a primal-dual active set strategy (PDASS), which has superlinear rate of convergence. To speed up the solution computation, we apply proper orthogonal decomposition (POD) method for model reduction and adjust the PDASS algorithm to the POD-Galerkin reduced problem. For that purpose we apply a-posteriori error estimation.
Joint work with Prof. Dr. Stefan Volkwein (University of Konstanz).

Economic model predictive control: closed-loop optimality and distributed implementation

Matthias A. Müller
University of Stuttgart, Stuttgart, Germany
matthias.mueller@ist.uni-stuttgart.de

Model predictive control (MPC) is an optimization-based control technology, which has found successful application in many different industrial fields. It consists of repeatedly solving a finite horizon optimal control problem and then applying the first part of the solution to the considered system. The main advantages of MPC and the reasons for its widespread success are that (i) satisfaction of hard input and state constraints for the closed- loop system can be guaranteed, (ii) optimization of some performance criterion can be directly incorporated in the controller design, and (iii) it can be applied to nonlinear systems with possibly multiple inputs.
In this talk, we focus on some recent developments in the MPC field, so called economic MPC schemes [1]. Here, in contrast to the classical control objective of stabilization, a more general performance criterion is considered which is possibly related to the economics of the considered system. In this case, the optimal operating behavior might not be stationary, but can be more complex (e.g. periodic). We discuss dissipativity conditions that guarantee both closed-loop performance bounds and convergence to the optimal operating behavior [2,3]. Furthermore, we consider the distributed implementation of economic MPC for large-scale systems. When using dual distributed optimization, which is scalable despite dynamic couplings between subsystems, one typically encounters primal constraint violations if the optimization is terminated after a finite number of steps due to real-time constraints. Based on a suitable constraint tightening similar to robust MPC, we present a method how such inexactness in the optimization can be taken into account when designing the economic MPC scheme, such that still closed-loop constraint satisfaction can be guaranteed. Finally, we briefly discuss an application case study, where economic MPC is used for economic dispatch in power networks [4].

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Some stability properties for a BDF2-type scheme for parabolic equations

Athena Picarelli
Mathematical Institute, University of Oxford
athena.picarelli@gmail.com

We consider a second order BDF (Backward Differentiation Formula) scheme for the numerical approximation of parabolic linear and nonlinear partial differential equations. The scheme under consideration is non monotone and second order accurate in time and space.
In the framework of Hamilton-Jacobi equations, the loss of monotonicity of the scheme prevents the use of the well known convergenece result of Barles and Souganidis [1].
However, recently applied to several optimal control problems (see [2]), this scheme has shown good performances and stability properties. Aim of this work is to analyse from the theoretical point of view these properties.

Joint work with O. Bokanowski (Université Paris 7) and C. Reisinger (University of Oxford).

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On economic model predictive control for time-varying systems

Simon Pirkelmann
University of Bayreuth, Bayreuth, Germany
simon.pirkelmann@uni-bayreuth.de

Model predictive control (MPC) can be seen as a method to approximate solutions to infinite horizon optimal control problems by iteratively solving problems on finite horizon. In classical MPC this is done with the aim of stabilizing the system at an equilibrium or tracking some reference trajectory at which the system performs optimal. In both cases the desired operating point or reference is assumed to be known a priori and enters the optimization problem through the stage cost.

Economic MPC presents an extension where the optimal reference no longer has to be known beforehand but is instead implicitly determined by the optimization. This becomes relevant when considering systems that depend on time-varying data. In this case the optimal reference will not simply be an equilibrium or periodic orbit but some more general time-varying trajectory that cannot be pre-computed.

Additional challenges arise due to the fact that for time-varying systems the cost functions may no longer yield finite values even for the optimal solution. This necessitates the use of the concept of overtaking optimality that allows us to treat such kind of problems.

In the talk we will investigate under which assumptions we can give performance estimates for the closed-loop cost of the economic MPC solution. We will see that a time-varying version of the turnpike property and continuity of the finite and infinite horizon optimal value functions allow us to prove that the cost of the MPC closed loop approximates the cost of an infinite horizon optimal trajectory. A simple example is given that illustrates the problems that occur and hints at the practical applications of the method.

This is joint work with Lars Grüne from the University of Bayreuth, Germany.

Optimal control of dynamical systems with sparse solutions

Zhiping Rao
Johann Radon Institute for Computational and Applied Mathematics
Austrian Academy of Sciences, Linz, Austria
zhiping.rao@ricam.oeaw.ac.at

In this talk, a class of infinite horizon optimal control problems involving L^p cost functionals with $0 < p \leq 1$ is discussed. The problem is convex when $p = 1$ and nonconvex when $0 < p < 1$. The existence of optimal controls is studied for the convex case, and it is also discussed in the framework of time-discretized model for the nonconvex case by extending the reparametrization approach introduced in [2]. The sparsity structure of the optimal controls promoted by the nonsmooth cost functional is analyzed. A dynamic programming approach is proposed to numerically approximate the corresponding sparse optimal controllers. This is joint work with Dante Kalise and Karl Kunisch (see [1]).

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Parametric Model Order Reduction for $\mathcal{H}_2$-Optimal Control Problems

A. Schmidt
University of Stuttgart, Stuttgart, Germany
andreas.schmidt@mathematik.uni-stuttgart.de

In this talk we consider the classical $\mathcal{H}_2$ optimal feedback control problem in a parametric and large-scale context. This setting allows for the solution of realistic control problems, since it considers disturbances in the system and in the measurement outputs. Furthermore it employs state-estimation techniques to reconstruct the unknown state from the noisy measurements. It turns out, that the controller is a dynamical system and two solutions of algebraic Riccati equations (AREs) are required to form it. We apply parametric model order reduction techniques to the AREs (cf. [1]) and to the state equation of the observer and show that this approach can yield a significant speed-up in multi-query scenarios for large scale parametric problems for the control of partial differential equations, see [2].
Joint work with B. Haasdonk (University of Stuttgart).

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Order reduction numerical methods for the algebraic Riccati equation

Valeria Simoncini
Alma Mater Studiorum, Università di Bologna, Italy
valeria.simoncini@unibo.it

In the numerical solution of the algebraic Riccati equation $A^*X + XA - XBB^*X + C^*C = 0$, where A is large, sparse and stable, and B, C have low rank, projection methods (see [3]) have recently emerged as a possible alternative to the more established Newton-Kleinman iteration. In spite of convincing numerical experiments (see, e.g., [1,4]), a systematic matrix analysis of this class of methods is still lacking. We derive new relations for the approximate solution, the residual and the error matrices, giving new insights into the role of the matrix $A - BB^*X$ and of its approximations in the numerical procedure [2].
In the context of linear-quadratic regulator problems, we show that the Riccati approximate solution is related to the optimal value of the reduced cost functional, thus completely justifying the projection method from a model order reduction point of view [2].

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Interpolation-based parametric model reduction for efficient damping optimization

Zoran Tomljanović
University of Osijek, Department of Mathematics, Osijek, Croatia
ztomljan@mathos.hr

We consider the optimization problem of semi-active damping of vibrating systems. The main problem is to determine the best damping matrix which will minimize the influence of the input to the output of the system. We use a minimization criteria based on the H_2 system norm. Since that the objective function is a non-convex function, this damping optimization problem usually requires a large number evaluations of objective function. Thus, we propose an optimization approach that calculates the 'interpolatory' reduced order model which allows significant acceleration of optimization process.

In our approach we use parametric model reduction based on Iterative Rational Krylov Algorithm, which ensures a good approximation of H_2 system norm. For the sampling parameters within the parametric model reduction we propose fixed sampling or adaptive sampling approach. Moreover, in order to preserve important system properties, we use second order structure, which in modal coordinates allows very efficient implementation of our approach.

Proposed approach provides efficient approximation of optimal parameters with significant acceleration of the optimization process, which is also illustrated in numerical experiments.

Joint work with Christopher Beattie and Serkan Gugercin (Department of Mathematics, Virginia Tech, USA).

Dynamic “factoring” techniques

Alex Vladimirsky
Cornell University, Ithaca, NY; USA
vladimirsky@cornell.edu

Theoretical results on accuracy of numerical schemes for differential equations are built on specific assumptions about the level of solution smoothness/regularity. But if a physically relevant solution is singular, this can severely degrade the convergence rates of standard numerical methods. When the exact location & type of singularity are known in advance, we can use the “factoring” techniques to circumvent this difficulty. The idea is to rewrite the original solution as a product (or a sum) of two functions: the first is chosen to have the exact right type of singularity at that location; the second is (at least locally) smooth but unknown and we recover it by solving a modified equation. We will illustrate this idea for ODE initial value problems and Eikonal PDEs with a point source. In the latter case, the “rarefaction fan” of characteristics yields a localized blow-up in second derivatives of the solution and decreases the rate of convergence even for simple (first-order upwind) discretizations.
However, rarefaction fans can also result from general (inhomogeneous) boundary conditions or discontinuities in coefficients of the equation. This talk will present a method for “dynamic factoring” in 2-dimensional Eikonal problems. The goal is to treat rarefaction fans as they are discovered in the process of solving the PDE on the grid.
Joint work with Dongping Qi (SJTU-Cornell).