Workshop

Mean Field Games and Related Topics - 4

BOOK OF ABSTRACTS

Università di Roma "La Sapienza"

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Mean field games with congestion: weak solutions

Yves Achdou

We consider a class of systems of time dependent partial differential equations which arise in mean field type models with congestion. The systems couple a backward viscous Hamilton-Jacobi equation and a forward Kolmogorov equation with periodic boundary conditions in the state variable. Because of congestion and by contrast with simpler cases, the latter system can never be seen as the optimality conditions of an optimal control problem driven by a partial differential equation. The Hamiltonian vanishes as the density tends to infinity and may not even be defined in the regions where the density is zero. After giving a suitable definition of weak solutions, we prove the existence and uniqueness results of the latter under rather general assumptions. No restriction is made on the horizon.

This is a joint work with Alessio Porretta (Roma II).

A General Theory for Discrete-Time Mean-Field Games

Tamer Basar

In this plenary lecture, I will present a general theory for mean-field games formulated in discrete time, and with discounted infinite-horizon cost. I will cover both perfect local state and decentralized partial state information structures. The state space of each player is a locally compact Polish space, and at each time, the players are coupled through the empirical distribution of their states, which affects both the players individual costs as well as their state transition probabilities. I will first discuss the difficulties to be encountered in any attempt to obtain the exact Nash equilibrium (even if it exists) in such dynamic games with decentralized information, with a finite number of players. The mean-field approach offers a way out of this difficulty, which is the topic of this lecture. First focusing on the perfect local state information, and using the solution concept of Markov-Nash equilibrium (under which a policy is player-by-player optimal in the class of all Markov policies), I will show under some (precise) mild conditions the existence of a mean-field equilibrium in the infinite population limit. I will then show that the policy obtained from the mean-field equilibrium is approximately Markov-Nash when the number of players is sufficiently large. Following this, I will turn to the class of discrete-time partially observed mean-field games. Using the technique of converting the original partially observed stochastic control problem (which arises in the infinite population limit when a generic player faces a cloud of players) to a fully observed one on the belief space and the dynamic programming principle, I will establish the existence of Nash equilibria under mild technical conditions. I will again show, as in the perfect local state information case, that the mean-field equilibrium policy, when adopted by each player, forms an approximate Nash equilibrium for games with sufficiently many players. At the end I will offer some thoughts for extensions to dynamic stochastic games with risk-sensitive and time-average cost functions, and games with hierarchies.
(Based on joint work with Naci Saldi and Maxim Raginsky)

Partially Observed Mean Field Games with Applications in Finance

Peter Caines

Partially observed mean field game (PO MFG) theory studies systems with a population of asymptotically negligible minor agents and a major agent, where, in the general case, all minor agents partially observe their own and the major agent's state, and the major agent partially observes its own state. Subject to technical conditions, the existence of e-Nash equilibria are established, together with the corresponding individual agents' (best response) control laws which depend upon information states generated by nonlinear filters associated to each agent. The optimal execution problem in financial markets is formulated in terms of a population of high frequency traders (minor agents) together with an institutional (major) investor, all of whom are (i) coupled through the mean field inherent in the market, and (ii) have partial observations on the major agent's inventories; furthermore, each agent is assumed to have a utility function corresponding to the maximization of its terminal wealth, and the avoidance of both large execution prices and high trading rates. PO LQG MFG theory then gives the best rate of trading for each agent to maximize its utility function in the ϵ-Nash equilibrium sense.
Work with A.Kizilkale, N. Sen and D. Firoozi.

On the long time behavior of the master equation in Mean Field Games

Pierre Cardaliaguet

In a joint work with A. Porretta (Roma 2), we study the long time behavior of the master equation in Mean Field Games (MFG) theory. This equation is a kind of transport equation in the space of measures which contains the MFG systems as characteristics. We show that its solution converges to a (weak) solution of an ergodic master equation as time tends to infinity. The non standard difficulty consists in estimating a uniform regularity of the solution with respect to the measure variable. As a consequence, we can characterize the long time behavior of the solution to the MFG systems. In a similar way, we analyze the convergence, as the discount factor tends to 0, of the discounted MFG systems.

A Semi-Lagrangian scheme for non linear Fokker-Planck equations and applications

Elisabetta Carlini

We propose a Semi-Lagrangian scheme for non linear Fokker-Planck equations. The scheme is first order, explicit, preserves non-negativity, conserves the mass and allows large time steps. The scheme is written in general dimension, as typical for Semi-Lagrangian scheme, it is free-mesh and it can be applied to solve linear and non linear-non-local problems. We prove a convergence result to the weak solution of the equation. In particular, we apply the scheme to solve a new Hughes-type model, for which we prove an existence result by applying the convergence analysis. The scheme can also be applied to solve Mean Field Games problems, when the coupling is non local.

The convergence results apply in dimension 1 (for weak solution) in general dimension (for smooth solutions). We show numerical simulations of a Lotka-Volterra model, of our new Hughes-type model and of a Mean Field Game model.
Joint work with Francisco Silva, Université de Limoges, France.

Synchronization of Circadian Rhythms: an MFG Model of Jet Lag

René Carmona

Ground states of focusing Mean Field Games on R^N

Marco Cirant

We consider stationary viscous Mean Field Games systems in the case of local and decreasing coupling: in this setting, agents prefer clustering in high-density areas. We set the problem in the whole euclidean space, adding a trapping potential that forces agents to stay in bounded regions. Under suitable conditions on the growth at infinity of the Hamiltonian and the coupling function, we obtain existence of solutions-equilibria as minimisers of associated non-convex functionals. We then analyse the problem in the vanishing viscosity regime, showing that such equilibria tend to concentrate on minima of the potential. Moreover, up to a rescaling, they converge to a limit system, which is translation-invariant. Here, a key role is played by an ad-hoc version of the concentration-compactness method.
This is a joint work with Annalisa Cesaroni.

Examples of restoration of uniqueness in mean field games

François Delarue

I will focus on examples of mean field games for which a form of common noise may restore uniqueness of the equilibria, whilst the corresponding version of the game, but without common noise, is not uniquely solvable. I will start with a simple case with Gaussian equilibria; in that setting, I will show that it may suffice to plug a finite-dimensional common noise to restore uniqueness. Then, I will address more general cases, for which the equilibria are no longer Gaussian and for which the standard Lasry Lions monotonicity condition for uniqueness fails; in this framework, I will provide a suitable reformulation of the game that becomes uniquely solvable when it is forced by an infinite dimensional common noise.

Mean field games with absorption

Markus Fischer

We discuss a simple class of mean field games with absorbing boundary over a finite time horizon. In the corresponding N-player games, the evolution of players' states is described by a system of weakly interacting It equations with absorption on first exit from a bounded open set. Once a player exits, her/his contribution is removed from the empirical measure of the system. Players thus interact through a renormalized empirical measure. In the definition of solution to the mean field game, the renormalization appears in form of a conditional law. We justify our definition of solution in the usual way, that is, by showing that a solution of the mean field game induces approximate Nash equilibria for the N-player games with approximation error tending to zero as N tends to infinity. This convergence holds provided the diffusion coefficient is non-degenerate. The degenerate case is more delicate and gives rise to counter-examples.
Joint work with Luciano Campi (The London School of Economics and Political Science).

Mean-field Optimal Control Hierarchy

Massimo Fornasier

In this talk we model the role of a government of a large population as a mean field optimal control problem. Such control problems are constrained by a PDE of continuity-type, governing the dynamics of the probability distribution of the agent population. We show the existence of mean field optimal controls both in the stochastic and deterministic setting. We derive rigorously the first order optimality conditions useful for numerical computation of mean field optimal controls. We introduce a novel approximating hierarchy of sub-optimal controls based on a Boltzmann approach, whose computation requires a very moderate numerical complexity with respect to the one of the optimal control. We provide numerical experiments for models in opinion formation comparing the behavior of the control hierarchy

First-order, stationary mean-field games with congestion

Diogo A. Gomes

Mean-field games (MFGs) are models for large populations of competing rational agents that seek to optimize a suitable functional. In the case of congestion, this functional takes into account the difficulty of moving in high-density areas. Here, we study stationary MFGs with congestion with quadratic or power-like Hamiltonians. First, using explicit examples, we illustrate two main difficulties: the lack of classical solutions and the existence of areas with vanishing density. Our main contribution is a new variational formulation for MFGs with congestion. Thanks to it, we prove the existence and uniqueness of solutions. In one and two-dimensions, a more detailed analysis can be performed. Finally, we consider some applications to numerical methods.

A Linear Quadratic Mean Field Team with Mixed Players: The Two-Scale Variational Approach

Minyi Huang

This talk presents a linear-quadratic mean field team consisting of a major player and N minor players which cooperatively minimize a social cost. The players have both cost coupling and dynamic coupling where the latter causes the mean field to be responsive to the control perturbation of an individual. We combine the person-by-person optimality principle with consistent mean field approximations to obtain decentralized strategies for the N+1 players, which asymptotically achieve social optimality. The limiting optimal control problems in our solution are constructed by handling two-scale variational problems which are due to the mean field coupling in the dynamics.
(This talk is based on a joint work with Son Luu Nguyen).

From the master equation to mean field game limits, fluctuations, and large deviations

Daniel Lacker

One way to understand a mean field game (MFG) is through its "master equation," an infinite-dimensional PDE for the value function. A solution of this equation can be used, for instance, to construct a solution of the original mean field game or to prove convergence of n-player equilibria to the MFG. This talk shows how to use a sufficiently smooth solution to answer several open questions about the limit theory for MFGs. In particular, we derive for the first time a central limit theorem and a large deviations principle for the n-player empirical measure (in equilibrium). The key idea is to use the master equation to quantitatively relate the n-player equilibrium to a McKean-Vlasov system of interacting diffusions.
This is joint work with Francois Delarue and Kavita Ramanan.

Closing The Loop of Optimal Trading: a Mean Field Game of Controls

Charles-Albert Lehalle

This talk explains how to formulate the now classical problem of optimal liquidation (or optimal trading) inside a Mean Field Game (MFG). This is a noticeable change since usually mathematical frameworks focus on one large trader in front of a " background noise " (or " mean field "). In standard frameworks, the interactions between the large trader and the price are a temporary and a permanent market impact terms, the latter influencing the public price. Here the trader faces the uncertainty of fair price changes too but not only. He has to deal with price changes generated by other similar market participants, impacting the prices permanently too, and acting strategically. Our MFG formulation of this problem belongs to the class of " extended MFG ", we hence provide generic results to address these " MFG of controls ", before solving the one generated by the cost function of optimal trading. We provide a closed form formula of its solution, and address the case of " heterogenous preferences " (when each participant has a different risk aversion). Last but not least we give conditions under which participants do not need to instantaneously know the state of the whole system, but can " learn " it day after day, observing others' behaviors.

Min-LQG games and collective discrete choice problems

Roland Malhamé

We introduce a novel class of finite horizon linear quadratic Gaussian games involving distinct potential finite destination states, interpreted as discrete choices under social pressure. The model provides stylized interpretations of opinion swings in elections, the dynamics of discrete societal choices, as well as a framework for achieving communication constrained group decision making in micro-robotic based exploration. Two distinct cases are considered: (i) The zero noise or "deterministic" case where agents are initially randomly distributed over their range space; (ii) The fully stochastic case. Under mild technical conditions, the existence of epsilon-Nash equilibria is established in both cases although these equilibria may in general be multiple. The corresponding agent control strategies are of a decentralized nature and are characterized in each case by the fixed points of a specific finite dimensional operator. Individual agent destination choices are fixed at the outset in case (i), while by contrast, their probability distribution evolves randomly along trajectories in case (ii), with a deterministic limit for the complete population as the latter grows to infinity.
This is joint work with Rabih Salhab and Jerome Le Ny.

Mean Field Games on Networks

Claudio Marchi

We consider stationary MFG defined on a network. In this framework the transition conditions at the vertices play a crucial role: the ones here considered are based on the optimal control interpretation of the problem. First, we prove separately the well-posedness of each of the two equations composing the MFG system. After, we prove existence and uniqueness of the solution to the MFG system on network. Furthermore, we propose some numerical methods, proving the well-posedness and the converging of the scheme.
These are joint works with F. Camilli and S. Cacace.

Boltzmann-type optimal control and model predictive control

Lorenzo Pareschi

In this talk we introduce a Boltzmann-type setting for the optimal control of large particle systems. In particular, we construct the corresponding model predictive control hierarchy for a finite time horizon. We show that, under a suitable scaling limit, the Boltzmann problem converges towards the corresponding mean-field optimal control problem and similarly for the corresponding mean-field model predictive approximation. Stochastic particle approximations are derived which permit to tackle the curse of dimensionality and to perform efficient numerical simulations. Several examples from opinion dynamics to swarming models are presented.

New estimates on quadratic variational Mean Field Games via techniques from the JKO world

Filippo Santambrogio

An important class of MFG (those which are deterministic and have a local monotone coupling) has a convex variational structure, and can be written as the minimization of an action, composed by a kinetic energy and a congestion penalization, in the class of curves in the Wasserstein space. The discretization in time makes some variational problems of the following form appear:

$$minF(\rho) + W_2^2(\rho, \rho_{k+1}) + W_2^2(\rho, \rho_{k-1}),$$

where W_2 is the classical Wasserstein distance. These problems are very similar to those that we find in the so-called JKO (Jordan-Kinderlehrer-Otto) scheme for the time-discretization of gradient-flow PDEs. The only difference is the fact that there are two reference measures instead of one. Techniques which have been very efficient in the study of the JKO scheme, such as the so-called flow-interchange introduced by Matthes, McCann and Savaré, can be applied in this framework and allow to obtain estimates on the optimal solution ρ. In particular, one can obtain in some cases that ρ is L^∞, and I will explain why this very estimate is very important for the equivalence between minimizing the action and being an equilibrium. Also, I will show how to obtain improved regularity in the case density-constrained case that we already studied with Cardaliaguet and Mészàros.
This is joint work with my PhD student Hugo Lavenant

On the variational formulation of some stationary second order MFGs

Francisco Silva

In this talk we consider an extension of the work by A. Mészàros and myself ('15) dealing with variational stationary MFGs with density constraints. We consider general Hamiltonians, satisfying a suitable growth condition, and also rather general coupling terms. For systems with and without density constraints we establish the existence of solutions using a variational technique and we also improve some of our previous results.