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WORKSHOP ABSTRACTS

The vanishing viscosity limit for Hamilton-Jacobi equations on networks

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Keywords : Vanishing viscosity, topological graph, viscosity solution

We consider the vanishing viscosity approximation for Hamilton-Jacobi equations defined on a network. We prove that a solution of the elliptic approximation exists and, as the viscosity vanishes, it converges to the viscosity solution of the original problem.

On exit-time and infinite horizon optimal control problems with a vanishing Lagrangian

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Keywords : Optimal control, exit-time problems, regularization, viscosity solutions

We consider a class of asymptotic exit–time and infinite horizon control problems for nonlinear systems with possibly unbounded data and a non-negative vanishing Lagrangian, including both coercive and cheap control problems. In general, the associated PDE may have multiple solutions. Moreover, known regularity, stability and, in the non coercive case, also well-posedness properties do not hold. In this paper we obtain such properties and a uniqueness result under some explicit sufficient conditions, generalizing previous hypotheses in several ways. In particular, we exploit some recent results on asymptotic controllability with a cost obtained by Motta and Rampazzo [MR] and optimality principles for degenerate, non coercive Hamiltonians proved in [M], which generalize previous results due

to Soravia [S]. Without aiming to be exhaustive, for the uniqueness issue and for an insight into some applications included in our setting, as for instance the Fuller or the shape from shading problems, we refer also to [IR], [CS], [Ma], and the references therein.

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An Introduction to Optimal Control of Partial Differential Equations with Real-life Applications

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Keywords : Optimal control, partial differential equations, numerical methods, real-time, state constraints, fuel cell systems, laser welding, hypersonic aircraft

When analyzing mathematical models for complex dynamical systems, their analysis and numerical simulation is often only a first step. Thereafter, one often wishes to complete the analysis by an optimization step to exploit inherent degrees of freedom for optimizing a desired performance index with the dynamical system as side condition. This generally leads to optimization problems of extremely high complexity if the underlying system is described by (time dependent) partial differential equations (PDEs) or, more generally, by a system of partial differential algebraic equations (PDAEs).

In the talk we will report on some of the lastest achievements on the field of optimization with PDEs and exhibit the challenges we are facing and have to cope with to solve such tasks. [6]

In the introduction three problems from engineering sciences are addressed for motivation:

1. Hot cracking is a common risk in welding of aluminium alloys. According to a Russian patent this risk can be avoided by applying a so-called multi-beam laser welding technique. By two additional laser beams the thermal stress introduced by the main welding laser can be compensated, if the additional laser beams are optimally placed and sized while they must not melt on the material. Mathematically one obtains a semi-infinite optimization problem with PDE and inequality constraints. [7]

2. Future concepts for intercontinental flights of passenger aircraft envisage aircraft which are able to fligh at hypersonic speeds. Due to such high velocities the thermal heating of the aircraft is an issue which has to be taken into account. This multi-physics problem leads to an optimal control problem for a system of ordinary differential equations where the heating of the aircraft's body is modelled as a quasilinear heat equation with nonlinear boundary conditions. The temperature of the thermal protection system must be limited and plays the role of a state variable inequality in this ODE-PDE optimal control problem. [2, 14]

3. The main example, which will be discussed in more detail in the second part of the talk, is concerned with the optimal control of certain fuell cell systems for an environmentally friendly production of electricity. Reaction-advection equations, a heat equation, additional ordinary as well as algebraic and integro equations sum up to a coupled system of up to 28 PDAEs of extremely high complexity. The inflow data into the anode inlet, the input data for the catalytic burner, and the amount of fed back from the cathode outlet are the control variables of this system. [1, 8–12]

After this motivation an outline of the mathematical theory of optimal control problems [13] for one elliptic equation is given to depict the purpose of solving optimal control problems by first order necessary conditions. Thereafter two numerical concepts, namely *first optimize then discretize* and *first discretize then optimize* [4] are discussed with respect to their pros and cons as well as an overlook on the mathematical toolbox from the literature is given.

The main part of the talk then deals with the results for optimal load changes when applying the two aforementioned methodologies including a method for the practical realisation of the computed optimal solutions based on model reduction techniques. [5]

If time allows some recent striking new ideas for the solution of state-constrained optimal control problems are sketched which lead to a new kind of optimization problem where optimal control, shape- and topology optimization establish a connection. [3]

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Control problems in stratified structures

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Keywords : viscosity solutions, discontinuous Hamiltonians, control on manifolds

We consider the state variable space partitioned into two open disjoint sets plus a common boundary, assumed to be a manifold with suitable regularity, and nicknamed interface. Further, we provide each open region of a control system with controlled dynamics, cost and discount factor. The two systems are a priori separate, apart the discount factors that it is not restrictive to take equal. We glue the pieces by using Filippov operator to mend the discontinuity of dynamics on the interface, and take the corresponding convex combination of costs. We finally study the infinite horizon discounted problem for the integrated system in the whole space, and focus on the value function with the aim of characterizing it as the unique viscosity solution of an appropriate Hamilton-Jacobi-Bellman equation. It turns out that it is actually possible under suitable compatibility conditions of the two original systems on the interface. The interesting mathematical point being that Ishii' s theory, which is the reference frame for equations with discontinuous Hamiltonians, does not fully apply here, due to the fact that it is not well adapted to the dynamical setting for the subsolution part. We detect an essential dynamics governing the system amd employ it in the definition of the Hamiltonian.

Noncoercive Hamiltonians, absolute minimizers and Aronsson equation

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Keywords : L^∞ minimization, Aronsson equation, distance functions, Hamilton-Jacobi equations

Recent developments in the area of Calculus of Variations in L^∞ have been motivated by a broad range of applications to problems where one seeks to minimize functionals represented as an essential supremum (worst case analysis) rather than the average cost, including optimal control, optimal transport, weak KAM theory, differential geometry, and degenerate elliptic PDEs associated with level set convexity. I will discuss the notion of absolute minimizer and the corresponding Aronsson equation for a noncoercive Hamiltonian. We can extend the definition of absolutely minimizing functions (in a viscosity sense) for the minimization of the L^∞ norm of a Hamiltonian, within a class of locally Lipschitz continuous functions with respect to possibly noneuclidian metrics. A special but relevant case contained in our framework is that of Hamiltonians with a Carnot-Caratheodory metric structure determined by a family of vector fields, in particular the eikonal Hamiltonian and the corresponding anisotropic infinity-Laplace equation. In general open domains and with a prescribed continuous Dirichlet boundary condition, we prove the existence of an absolute minimizer and derive the Aronsson equation as a viscosity solution for such a minimizer.

Inward pointing trajectories, Lavrentieff phenomenon and normality of maximum principle for Bolza problem under state constraints

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Keywords : Normality, Bolza problem, Lavrienteff phenomenon

We consider the following Bolza problem under state and end-points constraints

$$\inf \left\{ \varphi(x(0), x(1)) + \int_0^1 L(t, x(t), u(t))dt \,\middle|\, x(\cdot) \in S^K_{[0,1]} \right\},$$

where $\varphi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ and $L : [0,1] \times \mathbb{R}^n \times \mathcal{Z} \to \mathbb{R}$ are given cost functions, $\mathcal{Z}$ a complete separable metric space, and $S^K_{[0,1]}$ is the set of all absolutely continuous solutions of

$$x'(t) = f(t, x(t), u(t)), \quad u(t) \in U(t) \quad \text{for a.e. } t \in [0,1],$$

satisfying the constraints $x(t) \in K$ for all $t \in [0,1]$ and $(x(0), x(1)) \in K_1$, where $U(\cdot)$ is a measurable set valued map from $[0,1]$ into nonempty closed subsets of $\mathcal{Z}$, $f : [0,1] \times \mathbb{R}^n \times \mathcal{Z} \to \mathbb{R}^n$, $f(\cdot, x, \cdot)$ is $\mathcal{L} \times \mathcal{B}$-measurable and $f(t, \cdot, u)$ is locally Lipschitz continuous, K and K_1 are closed subsets of $\mathbb{R}^n$ and $\mathbb{R}^n \times \mathbb{R}^n$ respectively.

It is well known that every strong local minimizer of the Bolza problem under state constraints satisfies a constrained maximum principle. In the absence of constraints qualifications the maximum principle may be abnormal, that is, not involving the cost functions. Normality of the maximum principle can be investigated by studying reachable sets of an associated linear system under linearized state constraints. We provide sufficient conditions for the existence of solutions to such system and apply them to guarantee the non occurrence of the Lavrentieff phenomenon in optimal control under state constraints.

The Fractional Optimal Control

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Keywords : fractional calculus, calculus of variations

Fractional calculus has its origin in the following question: can the meaning of derivatives of integer order n be extended to when n is any number (irrational, fractional or complex)? Recent developments in the fields of science, engineering, economics, bioengineering, and applied mathematics, have demonstrated that many phenomena in nature are modeled more accurately using fractional derivatives and integrals. In the last few years, several works have been dedicated to create and develop the fractional optimal control. The new theory has been used to develop fractional mechanics, and to model the dynamics of many physical systems. In this talk we present a personal view to the subject and some recent results. For more on the subject we refer to the recent book:

> Agnieszka B. Malinowska and Delfim F. M. Torres,
> Introduction to the fractional calculus of variations,
> Imperial College Press, London & World Scientific Publishing,
> Singapore (2012). ISBN 978-1-84816-966-1

Properties of Optimal Trajectories that are not also Relaxed Optimal Trajectories

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Keywords : optimal control, maximum principle, nonlinear systems

Relaxation is a widely used regularization procedure in optimal control, according to which velocity sets are replaced by their convex hulls, thereby ensuring the existence of a minimizer. It can be an important step in the construction of sub-optimal controls for the original, unrelaxed, optimal control problem (which may not have a minimizer), based on obtaining a minimizer for the relaxed problem and approximating it. In some cases the minimum cost of the unrelaxed problem is strictly greater than the minimum cost over relaxed state trajectories; we need to identify such situations since then the above procedure for constructing sub-optimal controls fails. We encounter also a breakdown of the Dynamic Programming method because, typically, solving the Hamilton Jacobi equation yields the minimum cost of the relaxed, not the original, optimal control problem. Following on from earlier work by Warga and Ioffe, we explore the relation between, on the one hand, non-coincidence of the minimum cost of the optimal control and its relaxation and, on the other hand, abnormality of necessary conditions (i.e. the validity of such conditions in a trivial form, in which the cost multiplier is set to zero).

Fast Model Predictive Contol and Moving Horizon Estimation for Tethered Planes

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Keywords : model predictive control, moving horizon estimation, multibody mechanical systems, high-index DAE, flight control

The Airborne Wind Energy paradigm proposes to generate energy by flying a tethered airfoil across the wind flow. An essential problem is posed by the control of the airfoil during the launching phase. One proposed strategy, the rotational startup, relies on a rotating platform to give the plane the necessary momentum to rise to the height, where the wind is strong enough.

Mechanical systems are typically characterized by fast-evolving dynamics. Those processes are often constrained, motivating for control and estimation approaches based on Nonlinear Model Predictive Control (NMPC) and Moving Horizon Estimation (MHE).

The control frequency of optimization-based techniques is limited by the symbolic complexity of the equations modeling the system. This complexity can often be dramatically reduced by using representations based on non-minimal coordinates, which result in index-3 differential-algebraic equations (DAEs).

To meet the real-time requirements, code generation of both algorithms is used, based on the Real Time Iteration (RTI) scheme and direct multiple shooting. The exported plain C-code is tailored to the model dynamics, resulting in computational times in the range of few milliseconds.

Numerical results will be shown, where MHE has been used also to estimate the wind speed without measuring it, resulting in an adaptive NMPC scheme.