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POSTER ABSTRACTS

Exponential mapping for the sub-Riemannian problem on the Engel group

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Keywords : sub–Riemannian geometry, optimal control, mobile robot, trailer, numerical computation

The nilpotent sub–Riemannian problem on the Engel group with growth vector $(2, 3, 4)$ is considered. The corresponding four-dimensional optimal control problem with two-dimensional control is stated as follows:

$$\dot{q} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v} \end{pmatrix} = u_1 \begin{pmatrix} 1 \\ 0 \\ -\frac{y}{2} \\ 0 \end{pmatrix} + u_2 \begin{pmatrix} 0 \\ 1 \\ \frac{x}{2} \\ \frac{x^2+y^2}{2} \end{pmatrix}, \quad q \in \mathbb{R}^4, \quad u \in \mathbb{R}^2,$$

$$q(0) = q_0 = (0, 0, 0, 0), \quad q(t_1) = q_1 = (x_1, y_1, z_1, v_1),$$

$$l = \int_0^{t_1} \sqrt{u_1^2 + u_2^2}\, dt \to \min .$$

This problem arises as a nilpotent approximation to nonholonomic systems in four-dimensional space with two-dimensional control, for instance for the system describing motion of a mobile robot with a trailer on a plane.

Exponential mapping, which parametrizes the family of all extremal trajectories for the problem, was computed. The global upper bound for the cut time was found in order to investigate the optimality question for extremal trajectories. The Maxwell strata corresponding to discrete symmetries of the exponential mapping determine decompositions of the preimage and the image of exponential mapping into such domains that restriction of the exponential mapping to these domains is a diffeomorphism. Thus the sub-Riemannian problem on the Engel group was reduced to solving systems of algebraic equations.

On the basis of these results, a software for numerical computation of a global solution to the sub-Riemannian problem on a group of Engel was developed. So solution of the path-planning problem for mobile trailer robot via nilpotent approximation will be developed.

Influence of boundary on the motility of micro-swimmers

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Keywords : Control theory, Biological and artificial micro-swimmers, Self propulsion, Movement and locomotion, Low-Reynolds-number, Stokes equation, Boundary effect.

Self-propulsion at low Reynolds number is a relevant problem in biological and biomedical sciences which is also of great interest from the mathematical point of view. Since the sixties, experiments proved that in confined geometries, microorganisms are attracted by the boundaries. We address the same problem (the influence of boundary on the motion of micro-swimmer) by using the control theory.
The results are applied on two specific swimmers which consist of N (here $N = 3, 4$) spheres connected by thin jacks which are able to elongate or shrink. The first results deal with the influence of a plane wall on the controllability of these particular micro-swimmers. We prove that a controllable swimmer remains controllable in a half-space whereas the reachable set of a non-fully controllable one is affected by the presence of a wall. The second study extends these controllability results in the presence of a rough wall.

Non-Lipschitz points and the SBV regularity of the minimum time function

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Keywords : Reachable sets, normal vectors, Maximum Principle, minimized Hamiltonian

This paper is devoted to the study of the Hausdorff dimension of the singular set of the minimum time function under controllability conditions which do not imply its Lipschitz continuity. We consider first the minimum time T to reach the origin for normal linear control systems with constant coefficients in $\mathbb{R}^N$. We characterize points around which T is not Lipschitz as those which can be reached from the origin by an optimal trajectory (of the reversed dynamics) with vanishing minimized Hamiltonian. Linearity permits an explicit representation of such set, that we call $\mathcal{S}$. Furthermore, we show that $\mathcal{S}$ is $\mathcal{H}^{N-1}$-rectifiable with positive $\mathcal{H}^{N-1}$-measure. Second, we consider a class of control-affine *planar* nonlinear systems satisfying a controllability condition: we characterize the set $\mathcal{S}$ in a neighborhood of the origin in a similar way and prove its $\mathcal{H}^1$-rectifiability. In both cases, T is known to have epigraph with positive reach, hence to be a locally BV function. Since the Cantor part of DT must be concentrated in $\mathcal{S}$, our analysis yields that T is SBV, i.e., the Cantor part of DT vanishes. Our results imply also that T is locally of class $\mathcal{C}^{1,1}$ outside a $\mathcal{H}^{N-1}$-rectifiable set. With small changes, our results are valid also in the case of multiple control input.

Extension of Chronological Calculus for Dynamical Systems on Manifolds

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Keywords : Chronological Calculus, geometric control, Lie brackets, Chow-Rashevskii Theorem

We present an extension of Chronological Calculus to the case of infinite-dimensional C^m-smooth manifolds. The original Chronological Calculus was developed by Agrachev and Gamkrelidze for the study of dynamical systems on C^∞-smooth finite-dimensional manifolds. The extension of this calculus allows for the study of control systems with merely measurable controls and may be applied to C^m-smooth manifolds modeled over Banach spaces. We apply our extension to establish a formula of Mauhart and Michor for the generation of Lie brackets of vector fields and we present a proof of the Chow-Rashevskii theorem on C^m-smooth manifolds modeled over Banach spaces.

Motion Planning Problem for some Control Systems Applied in Robotics

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Keywords : Nilpotent approximation, car with two trailers, rolling ball

The classic objects of study in robotics are mathemetical models of wheeled mobile robots and robots-manipulators. In general, such systems are described by nonlinear nonholonomic control system linear with respect to control

$$\dot{q} = \sum_{i=1}^{n} u_i(t) X_i(q), \tag{0.1}$$

where the state space $Q \ni q$ is a connected smooth manifold, the controls $(u_1, \dots, u_n) \in \mathbb{R}^n$ are measurable and locally bounded, and $X_1, \dots, X_n$ are smooth vector fields. An interesting case occurs when the dimension of the state space exceeds the dimension of control $\dim Q > n > 1$. In generic case the minimal dimension of control $n = 2$ generates a completely controllable system which can reach any desired configuration from any initial configuration. A two-point boundary value problem for such systems is studied. The problem also known as the motion planning problem. The aim is to find controls $(u_1(t), u_2(t))$ which transfer system (0.1) from any given initial state $q^0 \in Q$ to any given terminal state $q^1 \in Q$:

$$q(0) = q^0, \quad q(T) = q^1. \tag{0.2}$$

A method of approximate solution based on the nilpotent approximation is used. The general method is concretized for solving the motion planning problem for five-dimensional systems with two-dimensional control:

$$\begin{gathered} \dot{q} = u_1(t)X_1(q) + u_2(t)X_2(q), \quad (0.2), \\ \dim(Q) = 5, \quad \rho(q(T), q^1) < \epsilon, \end{gathered}$$

where ρ is a distance on manifold Q.

Specific systems of the type under consideration is the kinematic model of mobile robot with two trailers and the ball rolling on a plane without slipping or twisting.

Large time behavior of weakly coupled systems of first-order Hamilton-Jacobi equations

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Keywords : weakly coupled systems, large time behavior, Hamilton-Jacobi equations

The large time behavior of solutions of first-order Hamilton-Jacobi equations has been extensively investigated using both PDE methods and dynamical approaches by Barles, Fathi, Ishii, Namah, Roquejoffre, Souganidis etc.. We are interested in extending the existence results of single equations to systems of first-order Hamilton-Jacobi equations. In our work, we only use the PDE method to get some results in this problem.

Sensitivity analysis for relaxed optimal control problems with final-state constraints

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Keywords : Sensitivity analysis, relaxation of optimal control problems, Pontryagin multipliers.

In this study, we consider a family of relaxed optimal control problems with final-state constraints, parameterized by a perturbation variable θ. Our aim is to compute a second-order expansion of the value $V(\theta)$ of the problems, in the neighborhood of a reference value of θ, say $\overline{\theta}$.

We begin by linearizing the family of optimization problems, which provides us with a second-order upper estimate of V. Then, considering a strong sufficient second-order condition and using a decomposition principle, we obtain a second-order lower estimate of V and a first-order expansion of the solutions.

We consider bouded strong solutions, which local optimal solutions in an arbitrary large neighborhood of the control and in a small neighborhood (both for the $L^\infty-$distance) of the trajectory. We work in the framework of relaxed optimal control problems in which the control variable is a Young measure, ie, at each time, the control is a probability measure on some space U, like if we were able to use several controls simultaneaously.

For our problems, relaxing does not change the value of the problems and it allows a very convenient way of linearizing the problems. Moreover, it allows us to prove a metric regularity theorem directly from abstract results, and to prove the existence of solutions.

This is a joint work with J. Frédéric Bonnans (Inria-Saclay and CMAP, Ecole Polytechnique) and Oana Silvia Serea (University of Perpignan).

A fully discrete semi-Lagrangian scheme for a first order Mean Field Game problem

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Keywords : Mean Field Games problems, First order Hamilton-Jacobi-Bellman equations, Semi-Lagrangian schemes.

In this work we consider the following model first order Mean Field Game (MFG) system:

$$
\begin{array}{rcl}
-\partial_t v(x,t) + \frac{1}{2}|D_x v(x,t)|^2 & = & F(x,m(t)), \;\; \text{in } \mathbb{R}^d \times (0,T), \\
\partial_t m(x,t) - \operatorname{div}\Big(D_x v(x,t) m(x,t)\Big) & = & 0, \quad \text{in } \mathbb{R}^d \times (0,T), \\
v(x,T) = G(x,m(T)) \quad \text{for } x \in \mathbb{R}^d & , & m(0) = m_0 \in L^\infty(\mathbb{R}^d).
\end{array}
$$

The above equations have been introduced by J.M. Lasry and P. L. Lions in order to model a deterministic differential game with an infinite number of players, supposing that the players are indistinguishable and each one of them has a small influence on the overall system.

In this poster I review the semi-discrete in time scheme studied with F. Camilli and introduce the fully-discrete one, analyzed with E. Carlini. We set ρ, h for the discretization parameters in space and time respectively. For the fully discrete scheme we prove that it admits at least one solution $v^{\rho,h}$, $m^{\rho,h}$. Moreover, in dimension one, i.e. the state of each microscopic player belongs to $\mathbb{R}$, we are able to prove that every limit of $(v^{\rho,h}, m^{\rho,h})$ (there always exists at least one) is a solution of the MFG system. Finally, some numerical experiments will be displayed.

Optimal Global Warming Policies when Discount Rates are Endogenous

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Keywords : optimal control, endogenous discounting, global warming, multiple equilibria

Our aim is to study, in the framework of the standard externality problem 'global warming', the consequences of endogenous discounting on the qualitative behavior and compare this approach with alternatives such as exogenous or myopic discounting. The Stern report and the ensuing public and academic debate has made clear how crucial discounting is for evaluating anti-global warming measures. A frequently proposed solution, concerning the issue of not weighting the future consequences of today's decisions enough, is hyperbolic discounting, which in turn leads to the problem of time inconsistency. Our basic assumption is that discounting decreases as the damage increases. This means that decision makers become more patient when facing the environmental damages following their high consumption levels. We show how a small change from an exogenous to an endogenous discount rate can lead to radical and sometimes counterintuitive differences in the qualitative behavior. An endogenous discount rate always smaller than the exogenous one can imply unexpectedly high steady state damages. Moreover, it can even trigger multiple steady states. Since endogenous discounting makes the somehow far-fetched assumption that the decision maker realizes how his future rate of discount changes with the evolution, we also consider an alternative called 'myopic' discounting. In this version, the decision maker applies the usual discount factor but uses the rate determined endogenously by the state of the world.