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Mini-course: "Shape Optimization"

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This series of lectures on shape optimization will be mainly focused on the optimal design problem in elasticity. It is known, since the pioneer work of Tartar and Murat in the late 70's, that this problem – as well as its corresponding problem in conductivity – is ill posed and doesn't always admits solutions. If the goal is to construct numerical methods to address this optimal design problem, this illposedness will lead to algorithms showing instabilities and erratic behaviours.

Two ways are possible to slightly change the initial problem so that it may be solved in an efficient and reliable way by numerical methods. The first one consists in enlarging the set of admissible shapes and allow, as solutions, "generalized shapes" that may contain fine mixture of the initial material and void. This process is called "relaxation" and the main tool involved is the homogenization theory. It leads to the so called "homogenization method for shape optimization". It will be discussed in detail, both from theoretical and numerical point of views, including practical issues necessary to write a numerical code.

The second approach consists in restricting the set of admissible solutions. A few theorems prove the existence of solutions under further geometrical, topological or regularity constraints. The classical domain variation method is revisited by using the level set representation, that may be seen as a constraint on the set of admissible solutions. The main tool is here the shape derivative. The level set algorithm will be explained extensively through various applications and examples.

Outline of the course

Lecture 1: Introduction to shape optimization. Homogenization method I

- Basic principles of shape optimization.
- The "seminal example" of non existence of classical solutions in conduction.
- Periodic homogenization.
- Laminated composites.
- Optimal bounds.
- Relaxation of the shape optimization problem.

Lecture 2: Homogenization method II. Algorithm and numerical issues

- The homogenization method for shape optimization.
- Compliance minimization.
- Other objective functions.
- Numerical algorithm.
- Technical numerical issues.

Lecture 3: Domain variation. Level set method I

- Existence theory.
- Gradient of a functional defined on an open set.
- Shape derivative.
- Topological gradient.
- Level set method.

Lecture 4: Level set method II

- Compliance minimization.
- Other objective functions.
- A damage evolution model.
- Technical numerical issue.