

**Summer School 'Optimal Control of PDEs'
Cortona, Italy, July 12-17, 2010**

**Lectures on
'Numerical Methods for Optimal Control of PDEs'**

This series of lectures addresses numerical methods for the approximate solution of optimal control problems associated with elliptic and parabolic PDEs including distributed and boundary control. We consider the unconstrained case, constraints on the control and/or the state, and gradient constraints. Optimal control of variational inequalities by MPEC techniques will be covered as well.

The approximation is based on finite element discretizations with respect to shape regular triangulations of the computational domain. The numerical solution of the discretized problems focusses on solvers and a posteriori error estimators for mesh adaptation. In particular, we discuss gradient based methods, primal-dual active set strategies, non-smooth Newton methods, and interior-point methods. For the a posteriori error analysis, we consider residual-type estimators as well as the goal-oriented dual weighted approach.

Applications include optimal control problems in finance, life and material sciences.

C O N T E N T S

Lecture 1: Optimal control of elliptic problems I

1. Introduction
unconstrained problems, control constraints,
state constraints, gradient constraints
2. FE Discretization and Solvers
gradient methods, active set strategies,
non-smooth Newton methods, interior-point methods

Lecture 2: Optimal control of elliptic problems II

1. A Posteriori Error Estimators
residual-type estimators
goal-oriented dual weighted approach
2. Elliptic Variational Inequalities
stationary points, discretization and solvers,
a posteriori error estimation

Lecture 3: Optimal control of parabolic problems

1. Introduction
2. FE Discretization and Solvers
3. A Posteriori Error Estimation

Lecture 4: Applications

1. Optimal Control in Finance
European double barrier basket options
2. Optimal Control in Life Sciences
optimal diffeomorphic matching in bioimaging
3. Optimal Control in Material Sciences
induction hardening