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Mini-Course “Model Reduction Methods”

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Description

The focus of this mini-course is on model order reduction methods for large-scale systems. These systems typically arise when partial differential equations are solved using numerical methods such as the finite element method. Model order reduction techniques provide an efficient and reliable way of solving these problems in the many-query or real-time context, such as optimization, characterization, and control.

More specifically, we present reduced basis (RB) approximation and associated *a posteriori* error estimation for rapid and reliable solution of parametrized partial differential equations (PDEs). The focus is on rapidly convergent Galerkin approximations on a space spanned by ”snapshots” on the parametrically induced solution manifold; rigorous and sharp *a posteriori* error estimators for the outputs/quantities of interest; efficient selection of quasi-optimal samples in general parameter domains; and Offline-Online computational procedures for rapid calculation in the many-query and real-time contexts.

Specific topics include primal-dual approximation and *a posteriori* error estimation for coercive and non-coercive elliptic PDEs and parabolic PDEs; greedy and POD sampling procedures; calculation of coercivity and inf-sup constant lower bounds by the Successive Constraint Method; and the Empirical Interpolation Method for consideration of non-affine parameter dependence.

We consider a wide range of PDEs with numerous examples drawn from heat transfer, linear elasticity, and fluid dynamics. We discuss application of RB techniques to shape optimization, parameter optimization, and optimal control.

We shall present all the steps required for practical implementation of the method — from calculation of the requisite snapshots with a standard ”available” code to online evaluation of the resulting reduced basis approximation and associated error bounds.

A detailed list of references will be provided for further reading and more advanced topics.

Outline

- I. Lecture 1: Motivation, Coercive Elliptic Problems
  - 1. Introduction/Motivation
    - a. Notation and Examples
    - b. Goal/Relevance
  - 2. Elliptic Problems I (coercive, affine, compliant)
    - a. Problem Statement, Truth Approximation, Affine Representation
    - c. Reduced Basis Approximation
    - d. Offline-Online Computational Procedures
    - e. Sampling Strategies: POD, Greedy, ...
    - f. *A Posteriori* Error Estimation
- II. Lecture 2: Elliptic Problems II, Parabolic Problems
  - 1. Elliptic: General Outputs (noncompliant), Non-symmetric Forms
    - a. Dual Problem, *A Posteriori* Error Estimation
  - 2. Elliptic: Noncoercive Problems
  - 3. Parabolic Problems
    - a. Problem Statement, Reduced Basis Approximation
    - c. *A Posteriori* Error Estimation
    - d. Offline/Online Decomposition
    - e. POD/Greedy-Sampling
- III. Lecture 3: Non-affine Problems, SCM, Software
  - 1. Non-Affine Problems
    - a. Empirical Interpolation Method
  - 2. Successive Constraint Method (SCM)
  - 3. Summary on Software: RB@MIT
- IV. Lecture 4: Applied Talk
  - 1. Optimization & Optimal Control
    - a. Parameter Optimization, GMA Welding Process, Advection-Diffusion (Environmental and thermal)
  - 2. Shape Optimization
    - a. Cardiovascular geometries, Airfoils (potential and thermal flows), FSI