Calcolare $H_k(X, \mathbb{R})$

con $X = \mathbb{R}^3 \setminus S^1$

$X = U$ $\quad V = S^1$ ingrossato cioè  Toro pieno.

$S^1 \times D_\varepsilon \sim S^1$.

Allora abbiamo

$\mathbb{R}^3 = U \cup V \qquad U \cap V \sim T_2$.

Quindi

$$0 \to \overset{\mathbb{R}}{\overset{\shortparallel}{H_2(U \cap V)}} \to H_2(U) \oplus \overset{0}{H_2(V)} \to \overset{0}{\overset{\shortparallel}{H_2(\mathbb{R}^3)}}$$

$$0 \to \overset{\mathbb{R}^2}{H_1(U \cap V)} \to H_1(U) \oplus H_1(V) \to 0$$

$k > 2 \quad H_k(X, \mathbb{R}) = 0$

Allora $H_2(X) = \mathbb{R} \qquad H_0(X) = \mathbb{R}$

$H_1(X) = \mathbb{R}$

$\mathbb{P}^2(\mathbb{R})$

Sia $U = \mathbb{P}^2(\mathbb{R}) \setminus \{[0,0,1]\}$

$V = \{[X, Y, Z] / Z \neq 0\}$

Calcolare omologie a coefficienti reali di U e V.