

Workshop on B-stable ideals and nilpotent orbits

October 8-12, Roma, Italia

Opening

October 8, 10:15

Schedule of Talks

Mon:	10:30-11:30	11:30-12	12-13	15-16	16-17	
	Kumar	Coffee Break	Möseneder	Panyushev	Righi	
Tue:	10-11	11-11:30	11:30-12:30	14:30-15:30	15.30-16:30	
	Kostant	Coffee Break	Nishiyama	Carnovale	Baur	
Wed:	9-10	10-11	11-11:30	11:30-12:30	12.30-13	
	Kac	Goodwin	Coffee Break	Athanasiadis	Kallipoliti	
Thu:	9:30-10:30	10:30-11:15	11:15-11:45	11:45-12:45	15-16	16-17
	Sommers	Maffei	Coffee Break	Reiner	Cellini	Achar
Fri:	9-10	10-11	11-11:30	11:30-12:30	12.30-13:30	
	Suter	Yu	Coffee Break	Fenn	Yakimova	

Titles and Abstracts

Title: *Eigencone, saturation and Horn problems for symplectic and odd orthogonal groups*

Speaker: Kumar

Abstract: This is a joint work with P. Belkale. We consider the eigenvalue problem, intersection theory of homogenous spaces (in particular, the Horn problem) and the saturation problem for the symplectic and odd orthogonal groups. The classical embeddings of these groups in the special linear groups play an important

role. We deduce properties for these classical groups from the known properties for the special linear groups. The tangent space techniques play a crucial role. Another crucial ingredient is the relationship between the intersection theory of the homogeneous spaces for $\mathrm{Sp}(2n)$ and $\mathrm{SO}(2n + 1)$. We solve the Horn and saturation problems for these classical groups.

Title: *Twisted Dirac operators in affine setting*

Speaker: Möseneder Frajria

Abstract: The abelian ideals of a Borel subalgebra index the multiplets arising from the decomposition of a fermionic representation. In this talk we show how this decomposition can be achieved by computing the kernel of a twisted version of Kac-Todorov Dirac operator. This operator is the affine analog of Kostant's cubic Dirac operator. Applications to asymptotic dimensions, the very strange formula, and an affine version of Vogan's conjecture are also given.

Title: *Some conjectures and observations on ad-nilpotent ideals and B-invariant subspaces*

Speaker: Panyushev

Abstract: In my talk, I am going to discuss at least two topics:

- a) Orbits of antichains in Δ^+ and related posets;
- b) On normalisers of B -stable subspaces in the little adjoint representations.

Title: *Ad-nilpotent ideals of a parabolic subalgebra*

Speaker: Righi

Abstract: We extend the results of Cellini-Papi on the characterizations of nilpotent and abelian ideals of a Borel subalgebra to parabolic subalgebras of a simple Lie algebra. These characterizations are given in terms of elements of the affine Weyl group and faces of alcoves. In the case of a parabolic subalgebra of a classical Lie algebra, we give formulas for the number of these ideals.

Title: *Borel Abelian Ideals and a Lie Algebra Generalization of the Amitsur-Levitski Theorem*

Speaker: Kostant

Title: *Resolution of null fibers and closed orbits on partial flag varieties*

Speaker: Nishiyama

Abstract: Using the machinery of dual pairs, we construct double fibration maps from the space of quivers to the Lie algebra of indefinite unitary group $G_{\mathbf{R}} = U(n, n)$. They can be interpreted as moment maps, which turns out to be a

quotient map. We will discuss a resolution of null fibers, whose quotient is a conormal bundle over a $K_{\mathbb{C}}$ -closed orbit on various partial flag variety arising from (complex) maximal parabolic subgroups. We also discuss the structure of the nilpotent orbits and the related degenerate principal series representations. (This is a joint work with Peter Trapa.)

Title: *Characterizing spherical conjugacy classes by the Bruhat decomposition*

Speaker: Carnovale

Abstract: A homogeneous G -space is called spherical if it has a dense orbit for a Borel subgroup B , or, equivalently if it is parted in a finite number of B -orbits. We will provide two characterizations of spherical conjugacy classes of G in terms of the Bruhat decomposition.

Title: *Δ -filtered modules for a quasi-hereditary algebra and adjoint orbits in a nilpotent radical*

Speaker: Baur

Abstract: Joint work with Karin Erdmann (Oxford) and Alison Parker (Leeds). Consider a parabolic subgroup of a reductive group G (defined over an algebraically closed field K). By a theorem of Richardson (1974), it is well-known that the adjoint action of P on its nilpotent radical has an open dense orbit. In general, there is an infinite family of orbits, so the description of the P -orbits is a “wild” problem. In type A there exists a translation of this problem into a question of representation-type of a category of representations of a path algebra due to Hille and Röhrle (1999), namely the Auslander algebra of the truncated polynomials $k[X]/X^r$. In this talk we will present our approach to type D .

Title: *Superization, quantization, chiralization*

Speaker: Kac

Title: *Verma modules for finite W -algebras*

Speaker: Goodwin

Abstract: We outline a construction of Verma modules for finite W -algebras due to Brundan, G. and Kleshchev. We show that Verma modules have a simple head and each finite dimensional simple module occurs as one of these simple heads.

Title: *B -stable ideals, simplicial complexes and hyperplane arrangements*

Speaker: Athanasiadis

Abstract: B -stable ideals, cluster complexes and certain deformations of Coxeter arrangements are three different families of objects attached to a finite, crystallographic root system, which have striking enumerative similarities. This lecture will be an overview of results and open problems related to, or stemming from,

the combinatorics of these families of objects and some of their natural generalizations.

Title: *The absolute order on the symmetric group*

Speaker: Kallipoliti

Abstract: The absolute order is a natural partial order on a Coxeter group W . It can be viewed as an analogue of the weak order on W in which the role of the generating set of simple reflections in W is played by the set of all reflections in W . By use of a notion of constructibility for partially ordered sets, it is proved that the absolute order on the symmetric group is homotopy Cohen-Macaulay. This answers in part a question raised by V. Reiner and M. Wachs. The Euler characteristic of the order complex of the proper part of the absolute order on the symmetric group is also computed. This is joint work with C. Athanasiadis

Title: *Minimal points in cells*

Speaker: Sommers

Abstract: We will discuss a method to compute points of minimal length in the dominant Weyl chamber that satisfy a certain set of properties related to nilpotent orbits. A connection to geometric cells in the affine Weyl group (introduced by Lusztig) will be presented.

Title: *Quotients of symmetric varieties*

Speaker: Maffei

Abstract:

Title: *Cyclic sieving of noncrossing partitions for complex reflection groups*

Speaker: Reiner

Title: *TBA*

Speaker: Cellini

Title: *Perverse coherent sheaves and special pieces*

Speaker: Achar

Abstract: Perverse coherent sheaves, introduced by Deligne, are the objects of a certain abelian subcategory of the derived category of equivariant coherent sheaves. In many ways, they resemble the much better-known perverse (constructible) sheaves, and in recent years, they have found a number of uses in representation theory. I will discuss some general properties of this and related categories of sheaves, and I will discuss some of their applications. In particular, I will discuss an approach using perverse coherent sheaves to a conjecture of Lusztig on the geometry of "special pieces," which are certain subvarieties of the nilpotent cone. This is joint work with D. Sage.

Title: *Some insights from cluster categories*

Speaker: Suter

Abstract: Enumerative properties in the theory of B -stable ideals are by now well-known to be linked to the combinatorics of cluster categories. The aim of my talk is to report on some combinatorial aspects from the perspective of cluster categories.

Title: *Richardson elements in seaweed Lie algebras*

Speaker: Yu

Abstract:

A seaweed Lie algebra \mathfrak{q} in a reductive Lie algebra \mathfrak{g} is a Lie subalgebra which is the intersection of two parabolic subalgebras \mathfrak{p} and \mathfrak{p}' whose sum is \mathfrak{g} . These Lie algebras were introduced by V. Dergachev and A. Kirillov in the $\mathfrak{gl}(V)$ case (and by D. Panyushev in the general case) in view of giving new examples of index zero Lie algebras. The set of seaweed Lie algebras contains all parabolic subalgebras and their Levi factors. We shall discuss recent results concerning the following question raised independently by M. Duffo and D. Panyushev : Does there exist an element x in the nilpotent radical \mathfrak{n} of a seaweed Lie algebra \mathfrak{q} such that $[\mathfrak{q}, x] = \mathfrak{n}$?

Such an element will be called a Richardson element of \mathfrak{q} . In other words, we wish to know if we can generalise Richardson's dense orbit theorem to seaweed Lie algebras.

By using a quiver representation setting generalising the one used by T. Brüstle, L. Hille, C. Ringel and G. Röhrle for parabolic subalgebras, we show that Richardson elements exist for seaweed Lie algebras in $\mathfrak{gl}(V)$. We observe from explicit computations that this is also the case if the rank of \mathfrak{g} is at most 7. However, there exists a seaweed Lie algebra in the simple Lie algebra of type E_8 which does not have a Richardson element.

Title: *Generating Equivalence Classes of B-stable Ideals*

Speaker: Fenn

Abstract: Given two ideals, I and J , we want to determine when $G.I = G.J$ so that I and J correspond to the same nilpotent orbit, $O_I = O_J$, in the Lie algebra. If this is the case, we will call I and J orbitally equivalent. Additionally, if I and J are orbitally equivalent, we can refine this equivalence. Given a nilpotent element e in O_I , we define a smooth projective variety X_I . Then $A(e)$ acts on the irreducible components of X_I , so looking at the stabilizer of a component gives us a subgroup of $A(e)$, defined up to conjugacy. Call this subgroup K_I and define a new equivalence relation as follows. We say I and J are K -equivalent if $O_I = O_J$ and $K_I = K_J$.

We will look at a series of simple "moves", ways to move from one ideal to a smaller ideal, that may generate these equivalence relations. In addition, these moves give some insight into how the varieties X_I are related, in particular how their homologies are related. This last point is motivated by the paper "Homology of the Zero-Set of a Nilpotent Vector Field on a Flag Manifold" by De Concini, Lusztig, and Procesi.

Title: *The coadjoint null-cone of a centraliser*

Speaker: Yakimova

Abstract: Let \mathfrak{g} be a reductive Lie algebra and e a nilpotent element in \mathfrak{g} . We consider the coadjoint representation of the centraliser \mathfrak{g}_e . If \mathfrak{g} is of type A , then, as was shown in a joint paper with Panyushev and Premet, the null-cone in \mathfrak{g}_e^* is equidimensional. This leads to a nice description of the tangent cone at e of the nilpotent cone $N \subset \mathfrak{g}$. We will prove that the null-cone in \mathfrak{g}_e^* is equidimensional for the distinguished nilpotent elements in classical Lie algebras.