1) Equazioni cartesiane e vettoriali della retta passante per $A \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $B \equiv \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

2) Equazioni cartesiane e vettoriali del piano passante per $A \equiv \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $B \equiv \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $C \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

3) Equazioni cartesiane della retta passante per $P \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ed ortogonale al piano $x_1 = 0$

4) Equazioni del piano contenente la retta $\begin{cases} x_1 = 3 \\ x_2 + x_3 = -1 \end{cases}$ e passante per l'origine

5) Equazioni cartesiane del piano parallelo alle rette $\begin{cases} x_1 = 0 \\ x_2 = 1 \end{cases}$ $\begin{cases} x_2 = 0 \\ x_3 = 4 \end{cases}$ e passante per $P \equiv \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

6) Distanza tra il punto $P \equiv \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$ e il piano ortogonale alla retta $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ passante per $P \equiv \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

7) Calcolare $\begin{pmatrix} 15 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$.

8) Dimostrare che $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

9) Dimostrare che $\sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} = 2^n$

1)
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + t\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}\right)$$
$$= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{cases} x_1 = 1+t \\ x_2 = 1 \\ x_3 = 1-t \end{cases} \qquad \begin{cases} x_2 = 1 \\ x_1 + x_3 = 2 \end{cases}$$

2)
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right) + s\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$$
$$= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{cases} x_1 = 1+s \\ x_2 = 1+t+s \\ x_3 = -s \end{cases} \rightsquigarrow \quad x_1 + x_3 = 1$$

3) direzione $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow r: \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$$

4) Fascio di piani per la retta $\lambda(x_1-3) + \mu(x_2+x_3+1)$

passaggio per 0 $\quad -3\lambda + \mu = 0 \rightsquigarrow \begin{cases} \mu = 3 \\ \lambda = 1 \end{cases}$

$$x_1 + 3x_2 + 3x_3 = 0$$

5) Direzioni delle rette: $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow \quad x_2 = 2$$

6) piano: $x_1 + 2x_2 + 4x_3 + d = 0$

piano passa per $\begin{pmatrix}1\\2\\2\end{pmatrix}$ $\quad 1 + 4 + 8 + d = 0 \quad d = -13$

$$x_1 + 2x_2 + 4x_3 - 13 = 0$$

$$d = \frac{|-1 -2 -4 -13|}{\sqrt{1+4+16}} = \frac{20}{\sqrt{21}}$$

7) $\binom{15}{4} = \frac{15!}{4!\,11!} = \frac{15 \cdot \cancel{14}^{7} \cdot 13 \cdot \cancel{12}}{\cancel{4 \cdot 3} \cdot 2} = 1365$

$\binom{3}{-2} = 0 \;;\quad \binom{6}{4} = \frac{6!}{2!\,4!} = \frac{6 \cdot 5}{2} = 15$

8) $\sum_{i=1}^{1}$ $\quad n = 1 \quad 1 = 1^3 = \left(\frac{1(1+1)}{2}\right)^3 = 1$ vera per $n = 1$

$$\sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3 = \frac{n^2(n+1)^2}{4} + (n+1)^3 =$$

$$= (n+1)^2\left(\frac{n^2}{4} + n+1\right) = \frac{(n+1)^2(n^2+4n+4)}{4} = \frac{(n+1)^2(n+2)^2}{4}$$

9) $n = 1 \quad \sum_{k=0}^{1} \binom{1}{k} = 2$ vera perché $\binom{1}{0} = 1$, $\binom{1}{1} = 1$

$$\sum_{k=0}^{n+1} \binom{n+1}{k} = \sum_{k=0}^{n+1} \left(\binom{n}{k} + \binom{n}{k-1}\right) =$$

$$\sum_{k=0}^{n} \binom{n}{k} + \sum_{k=1}^{n+1} \binom{n}{k-1} = 2^n + \sum_{k=0}^{n} \binom{n}{k} =$$

$$= 2^n + 2^n = 2^{n+1}$$