① $\sum_{i=0}^{2} 3^{i-1} x^{2-i} = a$

$\frac{1}{3}x^2 + x + 3 = a$

$x^2 + 3x + 9 - 3a = 0$

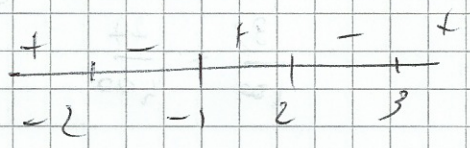
$9 - 4(9-3a) \geq 0$

$9 - 36 + 12a \geq 0$

$12a \geq 27 \qquad a \geq \frac{27}{12} = \frac{9}{4}$

②

$$\begin{cases} \frac{x^2-5x+6}{x^2+3x+2} \geq 0 \qquad \frac{(x-2)(x-3)}{(x+2)(x+1)} \\ x \geq 0 \end{cases}$$



$[0,2] \cup [3,+\infty)$

③ $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \frac{3}{2}$

$\lim_{x\to+\infty} \frac{5x^2(x-3x^2)^3}{(-2x^2-1)^n} = \lim_{x\to\infty} \frac{-27\cdot 5x^8 + \dots}{16\cdot x^8 + \dots} = \frac{-135}{16}$

$\lim_{x\to 0} \frac{\sin(x)}{e^x-1} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{x}{e^x-1} = 1$

$\frac{\sin x}{x} \to 1 \qquad \frac{x}{e^x-1} \to 1$

④ $y = mx + q$

$$\begin{cases} -1 = m+q \\ -3 = 2m+q \end{cases} \qquad \begin{cases} q = 1 \\ m = -2 \end{cases}$$

$\log_{10}(y) = -2x + 1$

$y = 10^{-2x+1}$

$\log_{10} y = -2\log_{10} x + 1$

$\log_{10} yx^2 = 1$

$y = \frac{10}{x^2}$

(5) $P(A) = \frac{2}{3}$ $P(B) = \frac{2}{3}$ $P(C) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{3} = P(A \cap C) = P(B \cap C)$

(6) $\frac{2}{11}$ / $\frac{1}{6}$ / $\frac{1}{6}$

(7) $f(x) = x - \cos x$ Dominio reale $\mathbb{R}$

$\lim_{x \to \pm\infty} f(x) = \pm\infty$

Asintoti obliqui $\lim_{x \to \infty} \frac{x - \cos x}{x} = 1$

$\lim_{x \to \infty} x - \cos x - x = \lim_{x \to \infty} -\cos x$ limite oscillante

$f'(x) = 1 + \operatorname{Sen} x$ $\operatorname{Sen} x = 1$ $x = \frac{\pi}{2} + 2k\pi$

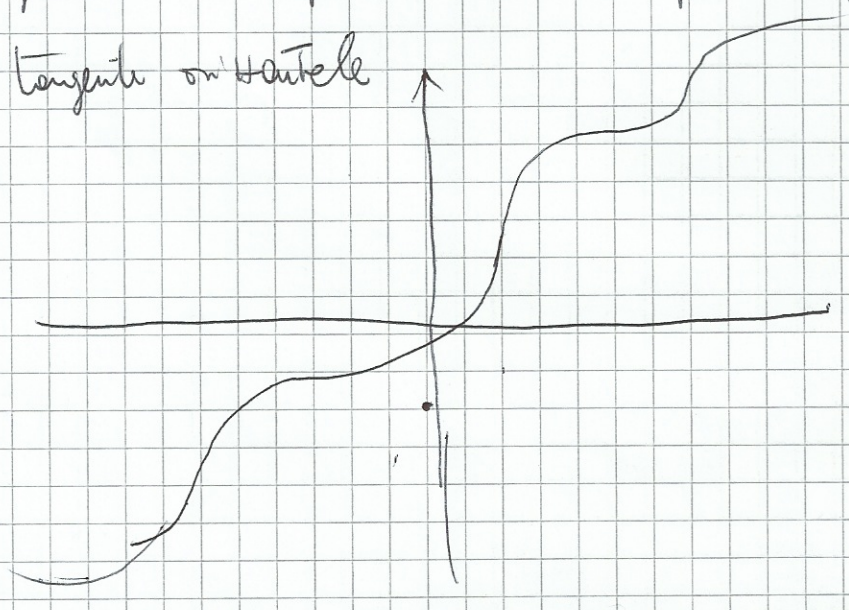
$f''(x) = \cos x \geq 0$

$f'(x) \geq 0$ $1 + \operatorname{Sen} x \geq 0$ $\operatorname{Sen} x \geq -1$ $\forall x$

$\Leftrightarrow$

$x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right]$

La funzione è sempre crescente e per $x = \frac{\pi}{2} + 2k\pi$ ha flesso a tangente orizzontale



punto di flesso

$x =$

(8)

C carnagione chiara

S carnagione scura

U ustionato

N non ustionat

$p(C) = \frac{11}{20}$

$p(S) = \frac{9}{20}$

$p(U|S) = \frac{3}{20}$ $\quad p(U|C) = \frac{3}{5}$

$$p(C|U) = \frac{p(U|C)\,p(C)}{p(U|C)\,p(C) + p(U|S)\,p(S)} = \frac{\frac{3}{5}\cdot\frac{11}{20}}{\frac{3}{5}\cdot\frac{11}{20} + \frac{3}{20}\cdot\frac{9}{20}} =$$

$$= \frac{\frac{33}{100}}{\frac{33}{100} + \frac{27}{400}} = \frac{33}{100}\cdot\frac{400}{159} = \frac{132}{159} = \frac{44}{53}$$

$p(NU|S) = 1 - p(U|S) = 1 - \frac{3}{20} = \frac{17}{20}$

$p(NU|C) = 1 - p(U|C) = 1 - \frac{3}{5} = \frac{2}{5}$

$p(S|U) = 1 - p(C|U) = 1 - \frac{132}{159} = \frac{27}{159} = \frac{9}{53}$