Quantitative results in stochastic homogenization

In many applications, one has to solve an elliptic equation with coefficients a that vary on a length scale much smaller than the typical scale of the right hand side f (or the size of the domain). We are interested in a situation where the field of coefficients a is characterized in statistical terms: Its statistics are assumed to be translation invariant and to decorrelate over large distances. As in the case of periodic homogenization, the solution operator behaves – on large scales – like the solution operator of an elliptic problem with *homogeneous, deterministic* coefficients. In other words, if the r. h. s. f has a typical scale that is large compared to the correlation length, then the solution u is close to the solution of a homogeneous, deterministic elliptic problem.

In this minicourse, we'll introduce a calculus that allows to estimate the speed of this convergence. We will focus on sharp estimates of the *random part* of the error, that is, we will estimate the fluctuations of $u(x)$ around its expected value.

To this purpose, we will consider the simplest possible setting, which is the one of a discrete elliptic equation, i. e. an elliptic equation on the d-dimensional lattice $\mathbb{Z}^d$. These estimates use tools from statistical mechanics (spectral gap, logarithmic Sobolev inequality) and elliptic regularity theory (De Giorgi, Nash). This is joint work with A. Gloria, S. Neukamm, and D. Marahrens.

In this minicourse, we will essentially follow [1]; the full error is estimated in [2], relying on results in [3]. The latter gives a more thorough introduction into the field of stochastic homogenization.

References

[1] Daniel Marahrens and Felix Otto, Annealed estimates on the Green's function, MPI-MIS preprint 69/2012

[2] Antoine Gloria, Stefan Neukamm, and Felix Otto, An optimal quantitative two-scale expansion in stochastic homogenization of discrete elliptic equations MPI-MIS preprint 41/2013

[3] Antoine Gloria, Stefan Neukamm, and Felix Otto, Quantification of ergodicity in stochastic homogenization: optimal bounds via spectral gap on Glauber dynamics — long version MPI-MIS preprint 3/2013