

RISPOSTE AGLI ESERCIZI DEL CAPITOLO 1 DELLE DISPENSE (per quelli con la doppia freccia si veda in fondo alle stesse).

1.2 i) 66 per cento; ii) 32 per cento; iii) circa il 48 per cento.

1.3 i) A Roma il 5 per cento, a Milano il 30 per cento; ii) a Roma il 2 per cento, a Milano il 3 per cento.

1.4 $a^{1/15}b^{1/2}c^{11/6}$, $a^{-5/12}b^{-17/12}c^{-13/6}$, $a^{1/15}b^{4/15}c^{-1/15}$.

1.5 $a^6b^3c^{3/2}$, $2a^{10/3}$, $\frac{3}{4}a^{-1/6}b^{-3/2}$, c .

1.6 $4^{30} < 8^{25} < 2^{200} < 16^{51}$.

1.7 $4 \log 2 + 7 - 11 \log 3 - 3 \log 7$, $\frac{1}{n}(12 \log a + p \log b - \log c)$, $\log 3$, $55 - 100$

1.8 $-\frac{3}{2} \log a + \frac{3}{2} \log b + 5 \log c$, $2 - 4 \log 5$, $\frac{1}{2}(\log a + \log b + \log c) - 2 \log 5 + 2$

1.9 45, 38, 87.

1.10 47, 95, 39.

1.11 90, 38.

1.12 47.

1.13 410 chilometri.

1.14 $\frac{x^3+x-1}{x^3+1}$, 0, $\frac{3x^2-2x+1}{(x^2-x+2)(x^2-1)}$, $\frac{-6x^2-9x-30}{2x^2-x-15}$, $\frac{-8x^2+42x-33}{(2x+3)(2x-3)^2}$, $\frac{-2x^2-x+1}{1-x^2}$, $\frac{1}{(1+x)^2}$, 0.

1.16 $a = -1$, $b = c = \frac{1}{2}$.

1.17 $x = \frac{5 \pm \sqrt{17}}{2}$, $x = \pm \sqrt{2}$, $x = \frac{1}{3}$, 3.

1.18 $x = \pm \frac{\sqrt{2}}{2}$, $x = 1$, impossibile.

1.19 $x - 1$, $\frac{1}{x-1}$, $\frac{2x+1}{x-1}$, $\frac{x^2-1}{2x}$.

1.20 $x = 1$, $x = 2$, $x = 2$.

1.21 $[-1, \frac{1}{2}]$, $(\frac{1-\sqrt{73}}{6}, \frac{1+\sqrt{73}}{6})$, tutti i reali, nessuna soluzione.

1.23 $(\frac{5}{2}, 4)$, $(-\frac{6}{7}, 3)$, $(-4, -3] \cup [1, +\infty)$.

1.25 $x = 2$, $x = 0$, -2 , $x = 1$, $-1 - \sqrt{2}$,

$x = \frac{-1-\sqrt{17}}{4}$, $x \geq 0$, $x = \frac{-3-\sqrt{17}}{2}$, 1, 2,

$x = 1$, $x = \frac{1-\sqrt{7}}{2}$, $x = \frac{\pm 7 \pm \sqrt{13}}{6}$,

$x = \frac{3+\sqrt{13}}{2}$, nessuna soluzione, $x = -2$, $-\frac{3}{2}$,

$x = -1 - \sqrt{2}$, 1, nessuna soluzione, $x = -\frac{4}{5}$, $\frac{4}{7}$.

1.26 $[-\frac{1}{2}, +\infty)$, $(-\infty, -1) \cup (5, +\infty)$, $[2, +\infty)$,

$[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}]$, $[\frac{-1-\sqrt{17}}{2}, +\infty)$, $(-\frac{1}{2}, 1)$,

$[-\frac{1}{2}, \frac{1}{2}]$, $[\frac{1}{2}, +\infty)$, $(-\infty, -1)$,

$(\frac{-3-\sqrt{33}}{2}, \frac{-3-\sqrt{17}}{2}) \cup (\frac{-3+\sqrt{17}}{2}, \frac{-3+\sqrt{33}}{2})$, $[-4, 1] \cup (6, +\infty)$, $[-1, 1]$.

1.27 $(-\infty, 0]$, nessuna soluzione, $(\frac{4}{5}, \frac{\sqrt{17}-1}{2})$,

nessuna soluzione, $(-3, -\frac{4}{7}) \cup (\frac{4}{7}, 3)$, $(-\infty, -2)$.

1.29 $x = \frac{5}{4}$, $x = \frac{13+\sqrt{13}}{2}$, nessuna soluzione, $x = \frac{1}{4}$, $x = \frac{5-\sqrt{13}}{2}$, $x = \frac{9+\sqrt{17}}{2}$.

1.30 Tutti i reali sono soluzione, $x = 2$, $x = \pm \frac{3}{4}$.

1.31 $(-\infty, -2)$, nessuna soluzione, $(2, +\infty)$,

$[-2, 1]$, $[1, +\infty)$, $[-\frac{5}{2}, 2]$,

$[-1, \frac{5}{4}]$, $[3, \frac{13+\sqrt{13}}{2}]$, $(-\infty, \frac{1}{3}]$.

1.32 $(-\infty, -1] \cup (1, +\infty)$, $(-\infty, -3) \cup (-2, 0)$, $(-3, 0]$, $(-\frac{9}{2}, \frac{5}{6})$, $(-\infty, 0) \cup (0, \frac{7}{3}) \cup (5, +\infty)$.

1.33 $(-\infty, -1] \cup (1, +\infty)$, $(-3, -2] \cup [0, +\infty)$, $[-1, 0) \cup [1, +\infty)$, $[-1, 1]$, $(-\infty, 1] \cup [2, +\infty)$,

tutti i reali eccetto 2, $(-\infty, 0] \cup (\frac{4}{3}, 3]$, tutti i reali eccetto 1 e 4, $(-3, +\infty)$,

$[-1 - \sqrt{11}, -1 + \sqrt{11}]$, $(-\infty, -2] \cup [\frac{2}{3}, +\infty)$, tutti i reali positivi, tutti i reali eccetto 3,

tutti i reali, tutti i reali eccetto -4 , tutti i reali eccetto $4 \pm \sqrt{6}$, $(-1, +\infty)$, $[0, +\infty)$.

CAPITOLO 3

3.1 0 minimo, 2 massimo; 0 estremo inferiore, 2 massimo; 1 minimo, 5 estremo superiore; $-\infty$ estremo inferiore (insieme illimitato inferiormente) e 5 estremo superiore; -1 minimo e $+\infty$ estremo superiore (insieme illimitato superiormente).

3.2 $f(0) = f(3) = \sqrt{2}$, $f(2) = f(3) = 0$.

3.4 Funzione non invertibile, per $y \in [-1, +\infty)$ (immagine) soluzioni $x = \pm\sqrt{1+y}$ per l'equazione $x^2 - 1 = y$; $\frac{13}{5y} + \frac{4}{5}$ per $y \neq 0$; $\frac{y+1}{y-1}$ per $y \neq 1$; $\frac{1}{2} - \frac{y}{12}$ per y reale qualsiasi; $\frac{2(y-7)}{y-5}$ per $y \neq 5$.

3.8 $-5, \mp\infty; 0, \pm\infty; -\frac{3}{2}, \mp\infty; 0, \pm\infty; \mp\infty, \frac{21}{4}; 0, \pm\infty; 1, \frac{6}{5}$.

3.9 $\frac{1}{4}, 2\sqrt{2}, \frac{4}{3}, \frac{1}{2}$, stesso del secondo.

3.11

$+\infty$ a $\pm\infty$, $-\infty$ a 0^- e $1+$; $+\infty$ a $\pm\infty$. $1/2$ a $\pm\infty$, $\pm\infty$ a $3/2\pm$; 1 a $\pm\infty$, 0 a 0^- , $+\infty$ a $0+$; $\pm\infty$ a $\pm\infty$, 0 a $0+$, $-\infty$ a 0^- (forma indeterminata);

e a $\pm\infty$, ∞ a -1^- , 0 a $-1+$. 1 a $\pm\infty$, 0 a 2^- , $+\infty$ a $2+$; 0 a $-\infty$, 1 a $+\infty$, $\pm\infty$ a $1\pm$; 0 a -1 , $+\infty$ a $+\infty$. 0 a $-\infty$, $+\infty$ a 1^- ;

1 a $\pm\infty$, $+\infty$ a -1^- , 0 a $0+$; 1 a $\pm\infty$, 0 a -1^- , $+\infty$ a $1+$; 0 a ± 1 ; $+\infty$ a ± 1 ;

0 a $\pm\infty$. $\mp\infty$ a $2\pm$; 0 a $\pm\infty$, $-1/2$ a -1 (singolarita eliminabile), $\pm\infty$ a $1\pm$; 0 a $-\infty$, $+\infty$ a $+\infty$, $\pm\infty$ a $2\pm$; $+\infty$ a $-\infty$, 0 a $+\infty$;

$-\infty$ a $-\infty$, 0 a $+\infty$; $\pm\infty$ a $\pm\infty$, 0 a $0+$, $-\infty$ a 0^- (forma indeterminata); $+\infty$ a $\pm\infty$, 0 a $0+$, $+\infty$ a 0^- (forma indeterminata); 0 a $-\infty$, $-\infty$ a 0^- ; $-\infty$ a $-2+$ e 2^- ;

0 a $0+$, $-\infty$ a $+\infty$; $+\infty$ a $2+$, $-\infty$ a $+\infty$; ± 1 a $\pm\infty$, $\pm\infty$ a $0\pm$; $+\infty$ a $-\infty$, 0 a $+\infty$; 0 a $-\infty$, $+\infty$ a $+\infty$;

0 a $\pm\infty$; 0 a $-\infty$, $+\infty$ a $+\infty$, $\pm\infty$ a $0\pm$; $+\infty$ a $\pm\infty$; $+\infty$ a $\pm\infty$, $-\infty$ a $-1/2^-$ e $0+$;

0 a $-1+$, $+\infty$ a $+\infty$; $\log 2$ a $1+$, $+\infty$ a $+\infty$; $+\infty$ a $-3/2+$, $-\infty$ a $+\infty$; 0 a $0+$, 0 a $+\infty$;

2 a $\pm\infty$, $+\infty$ a $0\pm$; e^{-1} a $\pm\infty$, 0 a 1^- , $+\infty$ a $1+$; $-\infty$ a $0+$, 0 a $+\infty$ (forma indeterminata); $\pm\infty$ a $\pm\infty$, 0 a 0^- , $-\infty$ a $0+$; 0 a $\pm\infty$, $+\infty$ a 0 .

3.14 $0, 0, 1, 1, 1, 1$.

3.15 $1, 1, e, e^a, \frac{7}{3}, 2$.

3.16 $-\infty, 0, 0, +\infty, \frac{\sqrt{2}}{2}, 0$.

CAPITOLO 4

4.2 $-\frac{\pi}{x^2}, 0, \frac{-x-x\tan^2x-1+\tan x}{x^2}, 2\sin(x) + x\cos x,$

$\frac{1-\cos x-x\sin x}{(1-\cos x)^2}, \frac{2e^x}{(1-e^x)^2}, -\frac{\sin x+\cos x}{e^x}, \frac{\cos x-2x\sin x}{2\sqrt{x}},$

$\frac{a}{\cos^2(ax)}, -be^{-bx}, -\frac{2}{x^2\cos^2(2/x)}, 5\cos(5x), 2\cos^2(x) - 1, 2\sin x \cos x.$

4.3 $-\frac{2}{x^3}, -\frac{n}{x^{n+1}}, \frac{1}{3x^{2/3}}, \frac{1}{2\sqrt{x}}, \frac{2}{3x^{1/3}}, -\frac{1}{2x^{3/2}},$ uguale, $-\frac{1}{3x^{4/3}}, \frac{1}{2\sqrt{x-1}},$

$-\frac{1}{(2-3x)^{2/3}}, -\frac{1}{|x-1|\sqrt{x^2-1}}, \frac{2}{5x^{3/5}}, e^x(1+x), 2e^x(xe^x-1)(1+x), 2\sin x \cos x, 2^x \log 2,$

$(\sin x)^x(\log(\sin x) + x \cot x), -\frac{2x}{(2-3x^2)^2}, \frac{1}{2\sqrt{x(1-x)}}, \frac{2xe^x+10e^x-1}{(x+6)^2}, \frac{\sin x}{\cos^2 x \cos^2(1/\cos x)},$

$-\frac{1}{2x}, \frac{2}{1-x^2}, -\frac{1}{1+x^2}, \log x + 1, 2\frac{3-4x}{2x+5} - 4\log(2x+5),$

$1 + e^{1/x}(1 - \frac{1}{x}), \frac{e^x(x-1)}{x^2}, -\frac{e^{1/(x-1)}}{(x-1)^2}, 2x \log(2-6x) - \frac{3x^2}{1-3x}, -\frac{2e^{1/x}}{x^2(e^{1/x}+1)^2},$

$\frac{3x^2+\cos x}{2\sqrt{x^3+\sin x}}, -\frac{1}{\sqrt{1+x^2}}, \frac{2}{\sqrt{4x^2+1}}.$

4.6

$2, +\infty, 1$, non esiste ma non e (non indeterminato, il modulo tende a $+\infty$),

$0, +\infty, \frac{1}{2},$

$-\frac{1}{2}, 1, 2,$

$+\infty, -\infty, -\infty$ (non indeterminato), 1

1, 1, $-\infty$ (non indeterminato), uguale,
 1, -1 (non indeterminato), -1 , uguale,
 $\frac{1}{7}$, $\frac{1}{2}$ (non indeterminato), -2 , 3 ,
 $\frac{3}{2}$, -2 , 2 , 0 (a $+\infty$) e -1 (a $-\infty$, non indeterminato),
 1 (non indeterminato), $\frac{1}{2}$, $-\frac{1}{12}$, $-\infty$.

4.7

1, 0, 0, 0.

CAPITOLO 5

5.1 3π , 25 , 4 , 2 , $\frac{7}{2}$.

5.2 -3π , 8 , 2 , $-\frac{3}{2}$, $\frac{1}{2}$.

5.3 2π , $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$, $\frac{2\pi}{3} + \sqrt{3}$.

5.4 B.

5.5 $10^{21}x$, $\frac{x^2}{2\pi}$, e^4e^x , $-10^{-12}e^{-x}$,

$18 \sin x$, $-\frac{\cos x}{2}$, $9 \arctan x$, $\frac{1}{4 \tan x}$,

$\sin(2x)$, $\frac{-7 \cos(2x)}{2}$, $\frac{2e^{5x}}{5}$, $\frac{5x^{25001}}{25001}$,

$\frac{x^3}{3} - \frac{x^2}{2}$, $\frac{x^2}{2} - \log|x|$, $\frac{3x^2}{2} - \frac{x^4}{4}$, $\frac{x^3}{3} - \frac{3e^{2x}}{2}$,

$x - e^{-x}$, $\log|x| - \frac{1}{x}$, $3 \sin x - \cos x$, $\frac{(x+1)^4}{4}$.

5.6 $\sin x + \cos x$, $\frac{2x^{5/2}}{5} - 2\sqrt{x}$, $\frac{2^x - 2^{-x}}{\log 2}$, $x - 2 \arctan x$.

5.7 $3x - \log|x+1|$, $x - 3 \log|x+3|$, $\frac{x}{3} + \frac{7}{9} \log|3x-1|$, $\log|x^2+x|$,

$e^{\sin x}$, $-\frac{e^{1-x^2}}{2}$, $\log|\log x|$, $\frac{(\log x)^2}{2}$,

$-\frac{3}{2} \cos(x^2+1)$, $\log|\sin(x^2)|$, $-\log|\cos x|$.

5.8 $-(x^2+2x+2)e^{-x}$, $\frac{x^2 \log x}{2} - \frac{x^2}{4}$, $x^2 \sin x + 2x \cos x - 2 \sin x$, $\frac{\sin(2x)}{4} - \frac{x \cos(2x)}{2}$.

5.9 $\frac{3 \sin(3x) \cos x - \cos(3x) \sin x}{8}$, $\frac{e^x(\sin x - \cos x)}{2}$, $-\frac{2^{-x}}{\log 2}(x - \frac{1}{\log 2})$, $\frac{x(\sin x)^2}{2} - \frac{x}{4} + \frac{\sin x \cos x}{4}$,

$\frac{e^{5x}}{5}(x - \frac{1}{5})$, $x(\log^2 x - 2 \log x + 2)$, $(\log x)^3 - 3x(\log x)^2 + 6x \log x - 6x$, $\frac{x^3}{3}(\log x - \frac{1}{3})$,

Uguale al precedente, $\frac{x^3(\log x)^3}{3} - \frac{2x^2 \log x}{9} + \frac{2x^3}{27}$, $-\frac{1}{x}(\log^2 + 2 \log x + 2)$, $\frac{e^x(\sin x + \cos x)}{2}$,

$\frac{2}{3}x(x+1)^{3/2} - \frac{4}{15}(x+1)^{5/2}$, $\frac{2}{3}x^2(x+1)^{3/2} - \frac{8}{15}x(x+1)^{5/2} + \frac{16}{105}(x+1)^{7/2}$, $x \log(1+x^2) - 2x + 2 \arctan x$, $\frac{1}{2}(x^2 \log(1+x^2) - x^2 + \log(1+x^2))$.

5.10 $\frac{1}{3} \sin(x^3)$, $\frac{1}{2} \arctan(x^2)$,

$\frac{1}{2}(\arctan x)^2$, $\frac{1}{48}(3x^2+1)^8$,

$2x^{1/2} + \frac{2}{3}x^{3/2}$, $\frac{2}{3}(e^x+1)^{3/2}$,

$e^{-x} - 2 \log(1+e^{-x})$, $\frac{2}{3}(1+e^x)^{2/3} - 2(1+e^x)$,

$\frac{1}{2}(\log(1+\sin x) + \log(1-\sin x))$.

5.11 $\log(x + \sqrt{x^2 - 1})$

5.12 $-\log(x + \sqrt{x^2 - 1}) + \sqrt{x^2 - 1}$, $\log(x + \sqrt{x^2 - 1}) + \sqrt{x^2 - 1}$.

5.13 $\frac{1}{16}(2x-3)^2 + \frac{3}{4}(2x-3) + \frac{9}{8} \log|2x-3|$, $\frac{1}{16}(4x-1 + \log|4x-1|)$, $\frac{1}{2}(4x-1)^2 - 6(4x-1) + 9 \log|4x-1|$, $\log|x+1| + \frac{4}{x+1}$,

$\frac{1}{2}(x-3)^2 + 6(x-3) + 10 \log|x-3|$, $\log|x-1| - \frac{1}{x-1}$, $\log|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2}$, $-\frac{1}{2(x+1)^2} + \frac{2}{3(x+1)^3}$.

5.14 $\frac{3}{4} \log|x+7| + \frac{1}{4} \log|x-1|$, $x - \frac{9}{4} \log|x+3| + \frac{1}{4} \log|x-1|$, $\frac{1}{3}(4 \log|x-2| - \log|x+1|)$, $3 \log|x+3| - 2 \log|x-2|$, $\frac{1}{4}(\log|x-2| - \log|x+2|)$, $\frac{9}{2} \log|x+2| - \frac{3}{2} \log|x|$, $\frac{3}{2}(x+1)^2 - 6(x+1)^2 + 2 \log|x+1|$,

$\frac{(y+1)^4}{4} - 4(x+1)^3 + 3(x+1)^2 - 4(x+1) + \ln|x+1|$, $\log|x-1| + \log|x|$, $\frac{4}{\sqrt{41}}(\log|x - \frac{5+\sqrt{41}}{2}| - \log|x - \frac{5-\sqrt{41}}{2}|)$.

$$\begin{aligned}
& 5.15 \frac{x+\log(2+e^x)}{2}, \frac{1}{2}(3\log|x^2-1|-\log(x^2+1)), \\
& -2\sqrt{x+1}+\log|\sqrt{x+1}-1|-\log(\sqrt{x+1}+1), \log x-2\log|1+\log x|+\log|\log^2 x-1|, \\
& \frac{x}{6}-\frac{1}{3}\log(3+e^x)-\frac{1}{2}\log(2+e^x), \frac{1}{3}(\log(2+x^2)-\log(5+x^2)). \\
& 5.16 \frac{1}{2}(x^2-1)\log|x^2-1|-\frac{x^2-1}{2}, \log|e^x-e^{-x}|, -e^{-x}(x^3+3x^2+6x+6), x(\log^2 x-2\log x+2), \\
& \frac{2}{3}(x+1)^{3/2}-4\sqrt{x+1}, \frac{2}{3}x^{3/2}\log x-\frac{4}{9}x^{3/2}, -\frac{1}{3}\log|1-x^3|, \\
& -\log|\cos x|, \tan x-x, \frac{\tan x-x}{2}-\log|\cos x|. \\
& 5.17 \frac{1}{4}(\log|x-1|-\log|x+1|-\frac{1}{x-1}-\frac{1}{x+1}). \\
& 5.18 x\sqrt{1-\frac{1}{x}}+\frac{1}{2}\log|1-\sqrt{1-\frac{1}{x}}|+\frac{1}{2}\log|1+\sqrt{1-\frac{1}{x}}| \\
& 5.19 \frac{13}{3}, 2^{5/2}(3^{3/2}-1), \log 3-\log 2, \frac{20}{3}, 4-2\sqrt{e}, \\
& \log(4+e)-\log 5, \frac{1}{3}(11^{3/2}-3^{3/2}), \frac{\log 2}{2}, \frac{1}{\log 2}.
\end{aligned}$$