Advanced Topics in Analysis

Laurea Magistrale in Matematica & Matematica per le Applicazioni

Lecture diary 2023/2024

December 20, 2023

Lecture 1-2 (September 26, 2023) Introduction to the course. Preliminaries to the notion of viscosity sub and supersolution. Vanishing viscous approximation of first order HJ equations. Definition of viscosity sub and supersolution for fully nonlinear second order elliptic PDEs. Some equivalent choices of test functions. The key stability result of the notion of viscosity sub and super solution.

Lecture 3-4 (September 28, 2023 Viscosity solution theory for first order HJ equations: sub and superdifferentials and relation wtih C^1 sub and supertangents; definition of viscosity sub and supersolution via super and subdifferentials; properties of sub and superdifferentials.

Lecture 5-6 (October 3, 2023) Properties of sub and superdifferentials. 1dimensional examples and discontinuity points of the derivative of a viscosity solution to $H(x, u') = 0$ in Ω. Comparison principles for a first order HJ equation set on a bounded open set. First comparison principle: statement and some remarks on the assumptions.

Lecture 7-8 (October 5, 2023) Proof of the first comparison principle. Second comparison principle: statement and proof.

Lecture 9-10 (October 10, 2023) Third comparison principle: statement and two proofs. Remarks and extensions.

Lecture 11-12 (October 17, 2023) Comparison principles for a first order HJ equation set on the euclidean space $\mathbb{R}^N$.

Lecture 13-14 (October 19, 2023) Existence results. The case of coercive Hamiltonians on $\mathbb{R}^N$. Clarke generalized gradient in finite dimension.

Lecture 15-16 (October 24, 2023) Clarke generalized gradient in finite dimension. Convex Hamiltonians: equivalence of the notion of viscosity subsolution and almost everywhere subsolution.

Lecture 17-18 (October 26, 2023) Convex Hamiltonians: equivalence among four different notion of subsolution for locally Lipschitz functions; remarks on the stability of the notion of viscosity subsolution for the inf operator. The Dirichlet problem: existence result. Evolutive H-J equations (with autonomous Hamiltonians): notion of discontinuous viscosity sub and supersolution. Statement of the comparison principle.

Lecture 19-20 (October 31, 2023) Evolutive H-J equations (with autonomous Hamiltonians): statement and proof of the existence of solutions. Convex Analysis:

definition of convex function; convexity vs. continuity/Lipschitz continuity.

Lecture 21-22 (November 2, 2023) Subdifferentials of a convex function. Local minima of a convex function are global minima. The finite dimensional case: properties of the subdifferential.

Lecture 23-24 (November 7, 2023) Subdifferential of a convex function vs. Clarke's generalized gradient. Fenchel transform and its properties. Bi-polar conjugate. Fenchel's inequality. Relation between the strict convexity of H and the C^1 regularity of H^*.

Lecture 25-26 (November 9, 2023) The Lax-Oleinik formula and the value function: motivation of the formula. The value function is a viscosity solution of the associated evolutive H-J equation.

Lecture 27-28 (November 14, 2023) Lipschitz regularity of the value function.

Lecture 29-30 (November 16, 2023) Existence of Lagrangian minimizers. The length functional: the support function $\sigma_a(x, \cdot)$ of the set $Z_a(x) := \{ p \in \mathbb{R}^N : H(x,p) \leqslant a \}$ and its properties.

Lecture 31-32 (November 21, 2023) The intrinsic length functional. An absolutely continuous curve can be seen as a reparameterization of a Lipschitz curve.

Lecture 33-34 (November 23, 2023) Lipschitz regularity of Lagrangian minimizers

Lecture 35-36 (November 28, 2023) Homogenization of periodic Hamilton-Jacobi equations.

Lecture 37-38 (November 30, 2023) Homogenization of periodic Hamilton-Jacobi equations.

Lecture 39-40 (December 7, 2023) Weak KAM Theory: part 1.

Lecture 41-42 (December 12, 2023) Weak KAM Theory: part 1.

Lecture 41-42 (December 14, 2023) Weak KAM Theory: part 1.

Lecture 45-46 (December 19, 2023) Weak KAM Theory: part 2.

Lecture 47-48 (December 21, 2023) Weak KAM Theory: part 2.