ADVANCED TOPICS IN ANALYSIS

CORSO DI LAUREA MAGISTRALE IN MATEMATICA & MATEMATICA PER LE APPLICAZIONI

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ABSTRACT. The course aims at introducing the student to the theory of viscosity solutions for nonlinear PDE and to some themes that are object of current research. After a general introduction to the viscosity solution theory, we will focus on the case of first-order Hamilton-Jacobi (HJ) equations with convex Hamiltonian and present the metric, variational and dynamical aspects of those equations. We will exploit this point of view to perform a qualitative study of a stationary Hamilton-Jacobi equation of eikonal type posed on a closed manifold (compact and without boundary). The tools and results we will present are the body of what is known as weak KAM Theory. This theory is relevant for the study of periodic homogenization of Hamilton-Jacobi equations. This topic will be presented and discussed in the last part of the course.

Details on the course: 48 hours, 6 CFU

Program:

1. Viscosity solution: definition and main properties
2. First order HJ equations
   a) comparison principles;
   b) existence and uniqueness results;
   c) Hamiltonians convex in the momentum;
   d) metric methods for eikonal-type HJ equations;
   e) time dependent HJ equations.
3. HJ equations and weak KAM Theory
4. Periodic homogenization of HJ equations

Evaluation method: the exam will be divided in two parts: in the first one, the student will be asked to answer to some questions on the first half of the course (items 1) and 2) above); the second part consists on an oral presentation of a topic related to the content of the course.

References: basic references for the theory of viscosity solutions are listed below. Further material will be made available during the course.

1. M. BARDI AND I. CAPUZZO-DOLCETTA, *Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations*, Systems & Control: Foundations & Applications, Birkhäuser Boston Inc., Boston, MA, 1997. With appendices by Maurizio Falcone and Pierpaolo Soravia.

2. G. Barles, *Solutions de viscosité des équations de Hamilton-Jacobi*, vol. 17 of Mathématiques & Applications (Berlin) [Mathematics & Applications], Springer-Verlag, Paris, 1994.