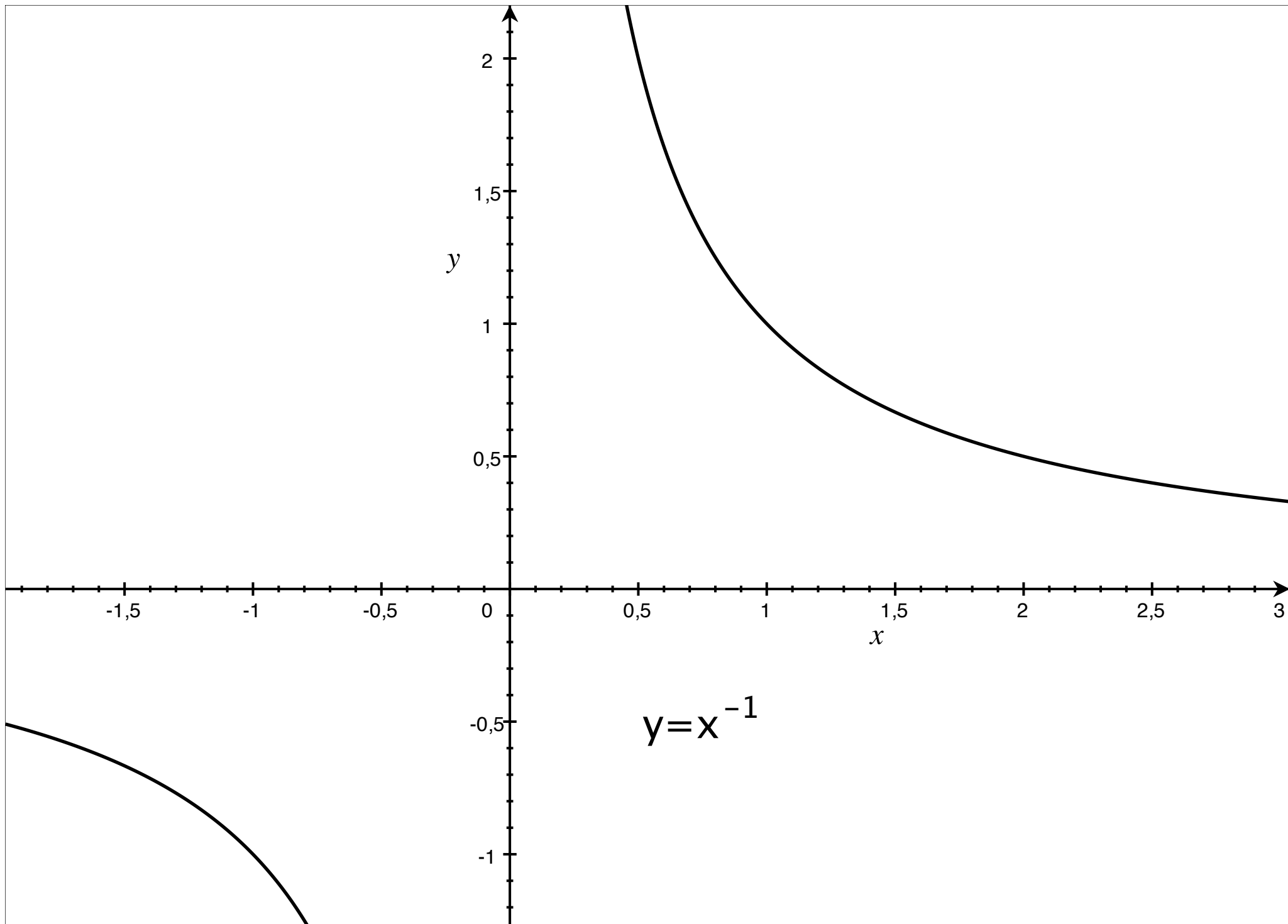
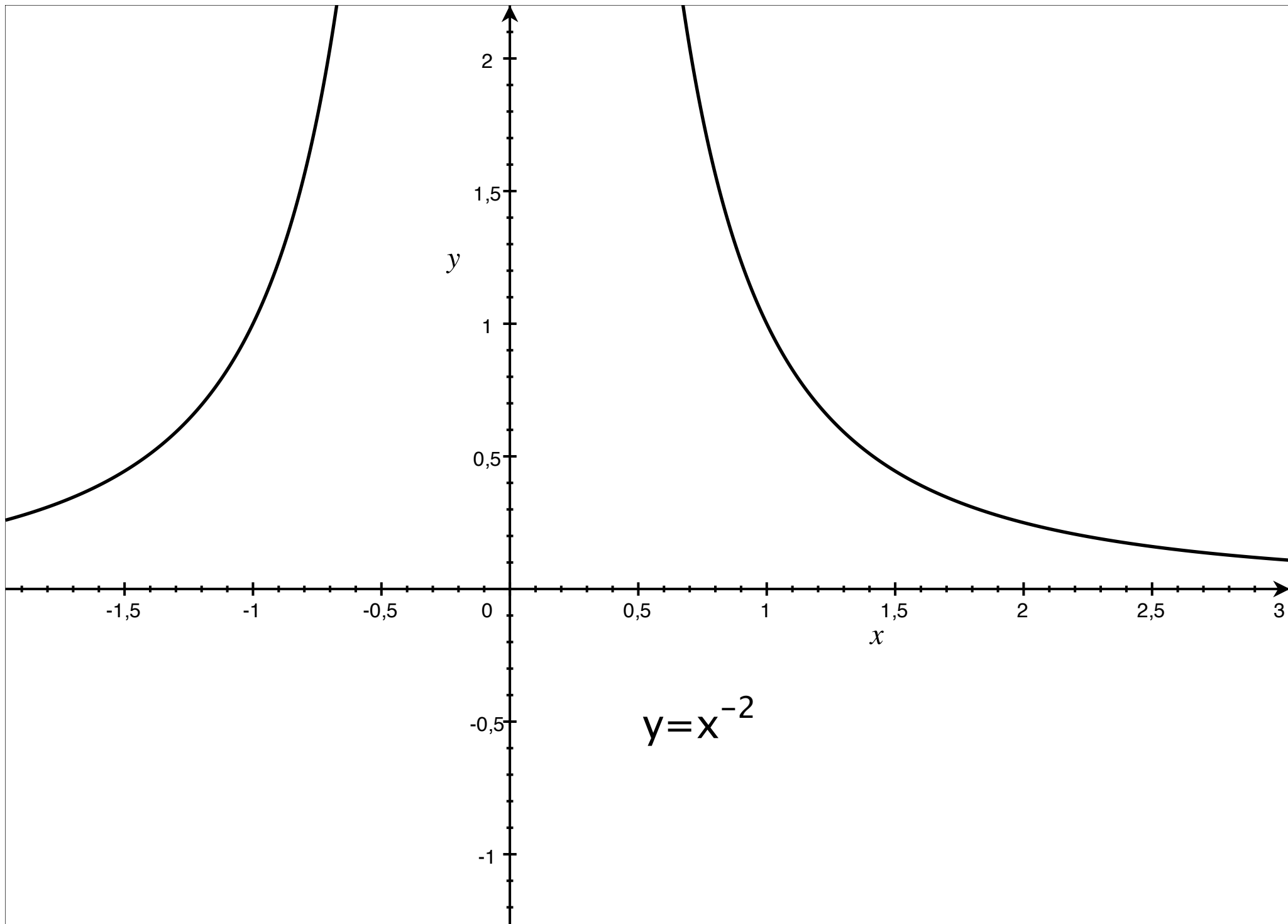


$$y = x^a$$

a = ...





Proprietà Per ogni $a, b \in \mathbb{R}^+$ e $r, s \in \mathbb{Q}$ risulta:

1) $a^{r+s} = a^r \cdot a^s$;

2) $(ab)^r = a^r \cdot b^r$;

3) $(a^r)^s = a^{rs}$;

4) $a^{-r} = \frac{1}{a^r}$;

5) $a^r > 0$, $a^0 = 1$, $1^r = 1$;

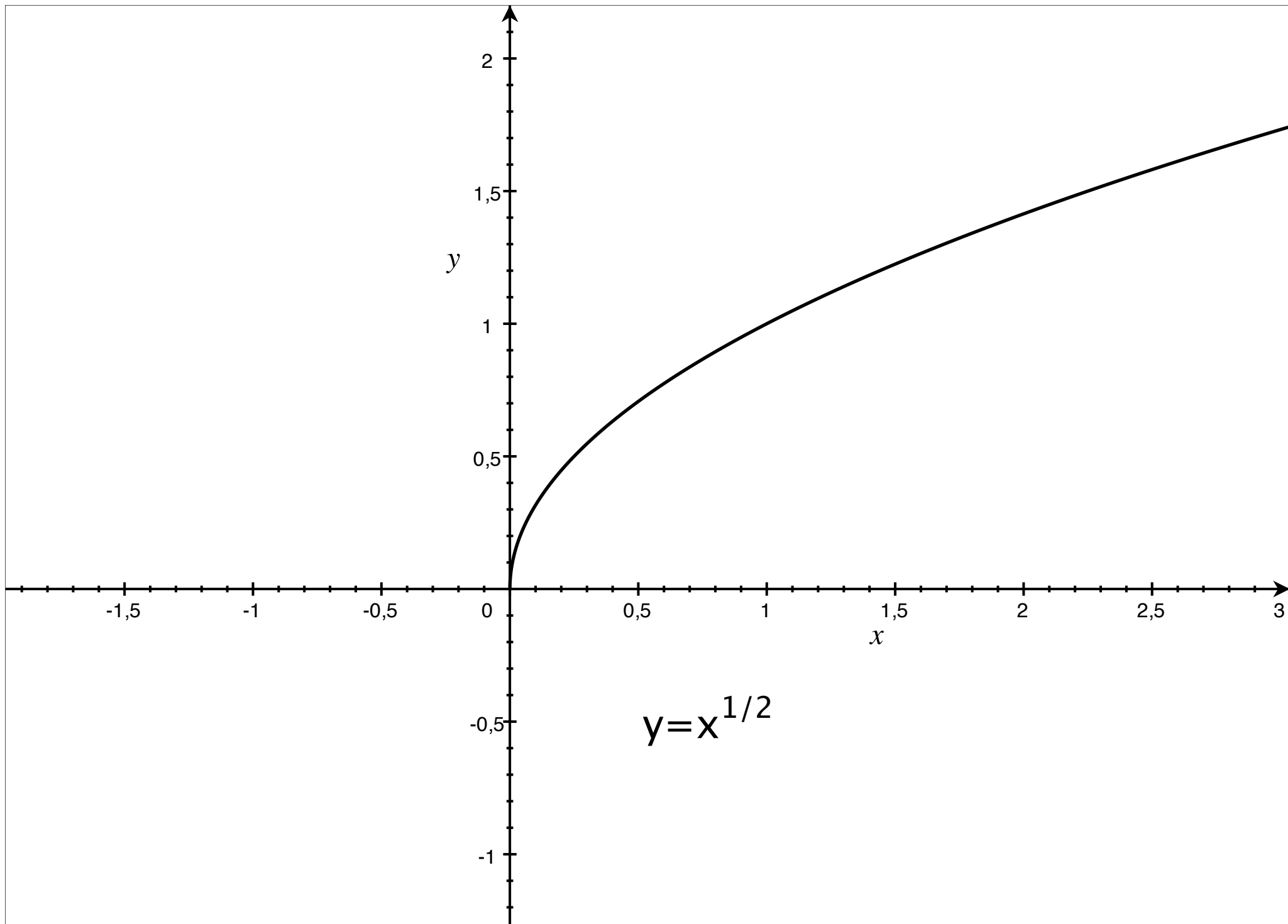
6) $\begin{cases} a^r > 1 & \text{se } a > 1 \text{ e } r > 0, \text{ oppure se } a < 1 \text{ e } r < 0 \\ a^r < 1 & \text{se } a < 1 \text{ e } r > 0, \text{ oppure se } a > 1 \text{ e } r < 0; \end{cases}$

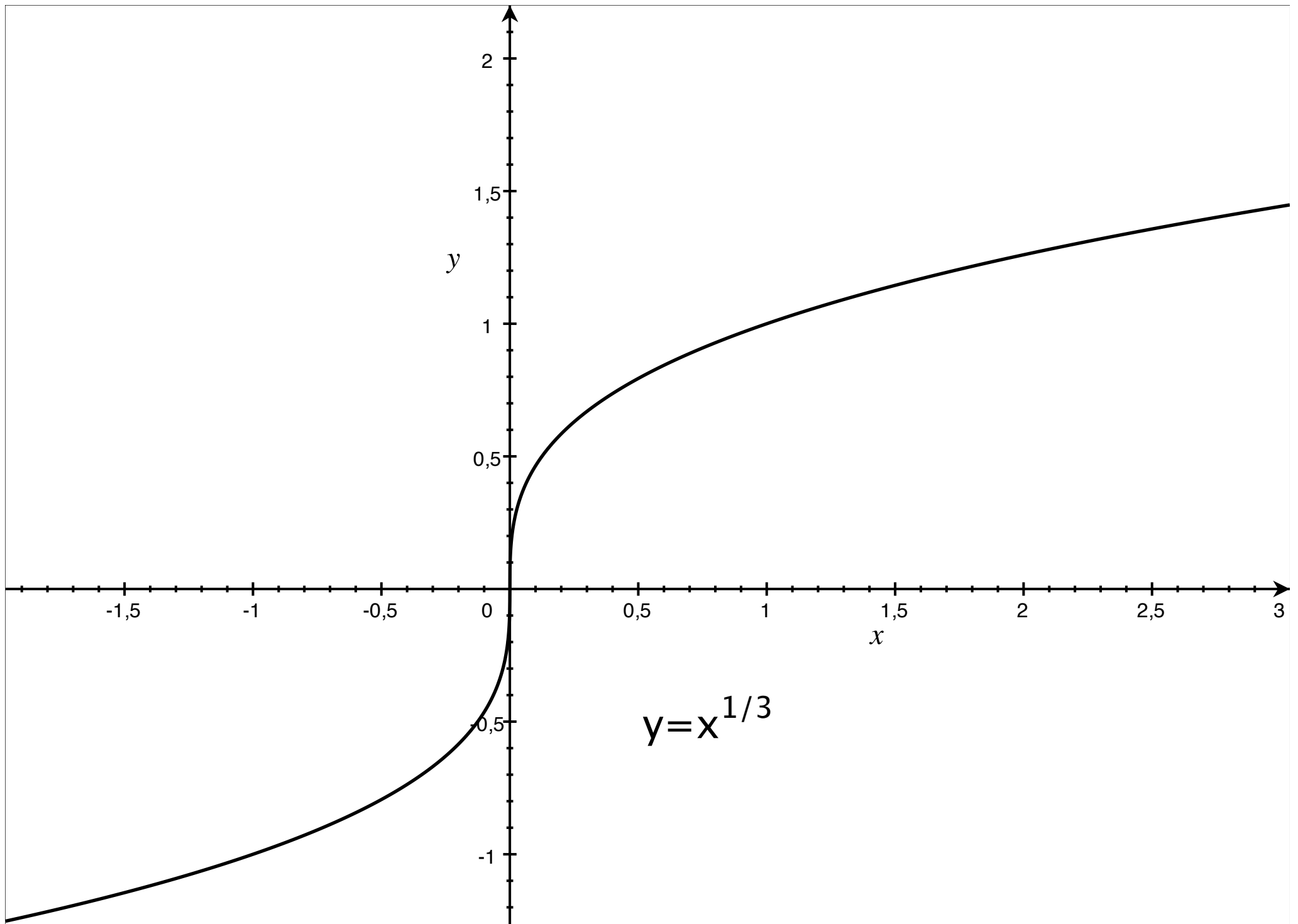
$$7) \quad r < s \Rightarrow \begin{cases} a^r < a^s & \text{se } a > 1 \\ a^r > a^s & \text{se } a < 1; \end{cases}$$

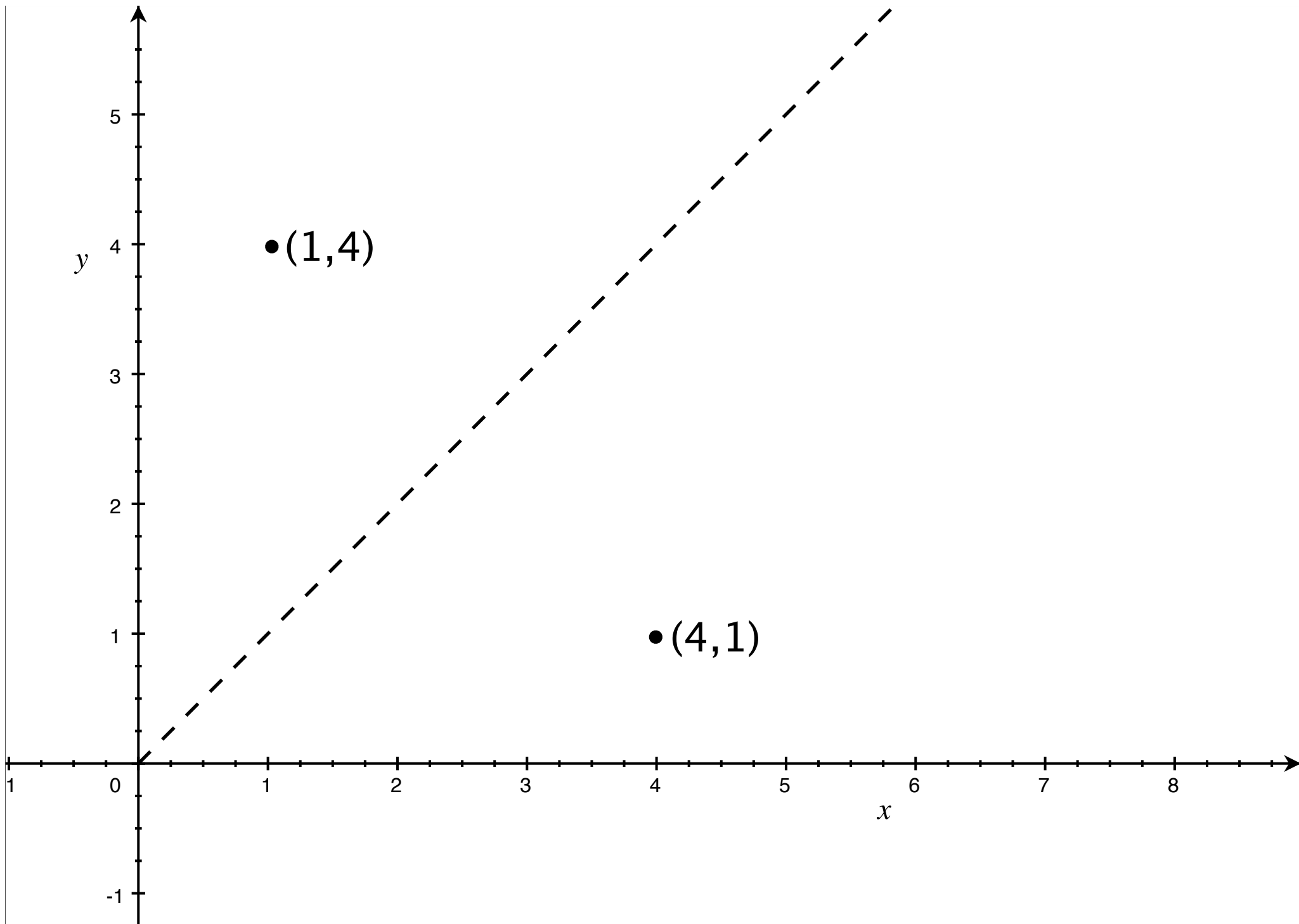
$$8) \quad 0 < a \leq b \Rightarrow \begin{cases} a^r \leq b^r & \text{se } r > 0 \\ a^r \geq b^r & \text{se } r < 0 \end{cases}$$

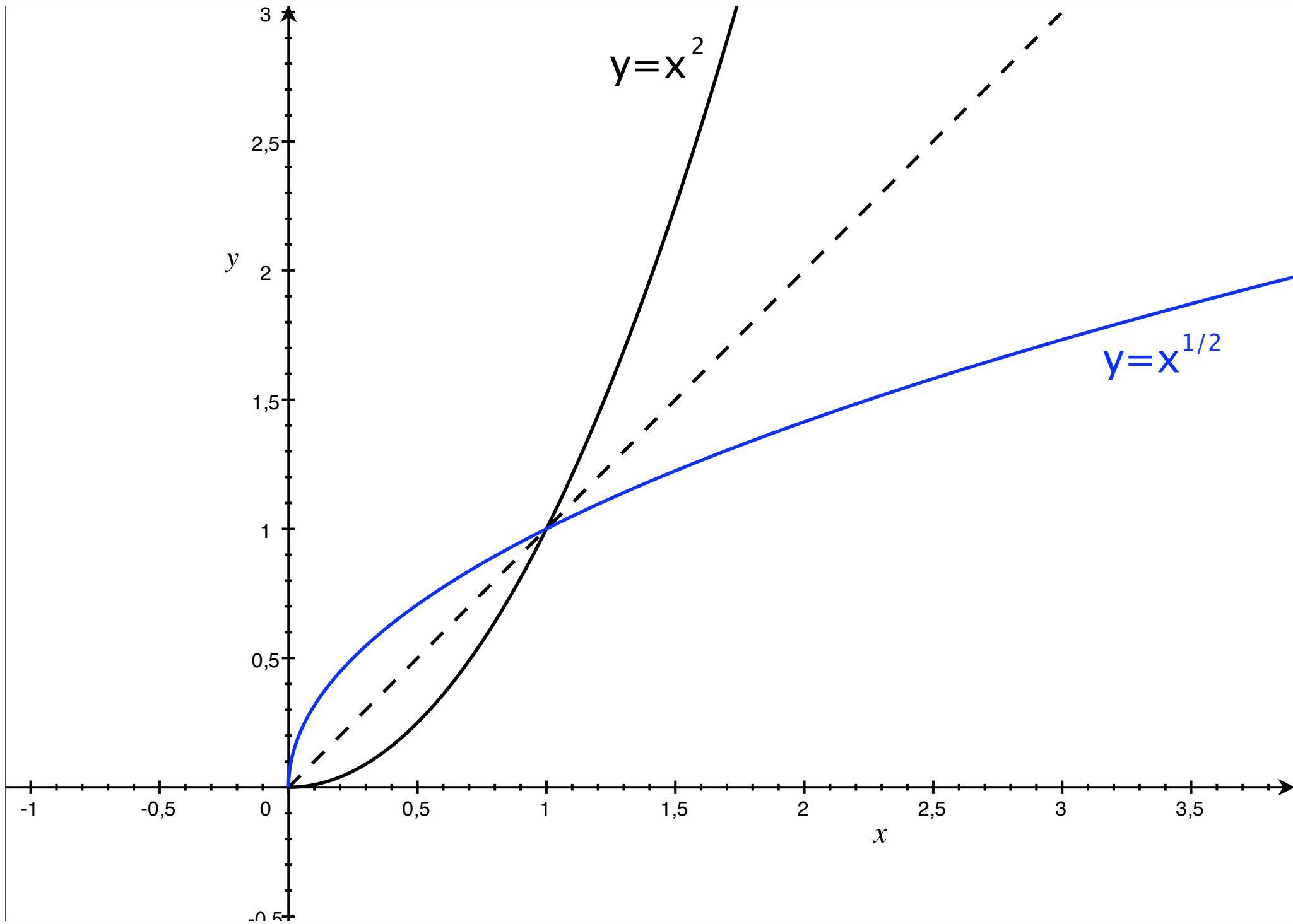
$$9) \quad \forall a \neq 1: \quad a^r = a^s \Rightarrow r = s$$

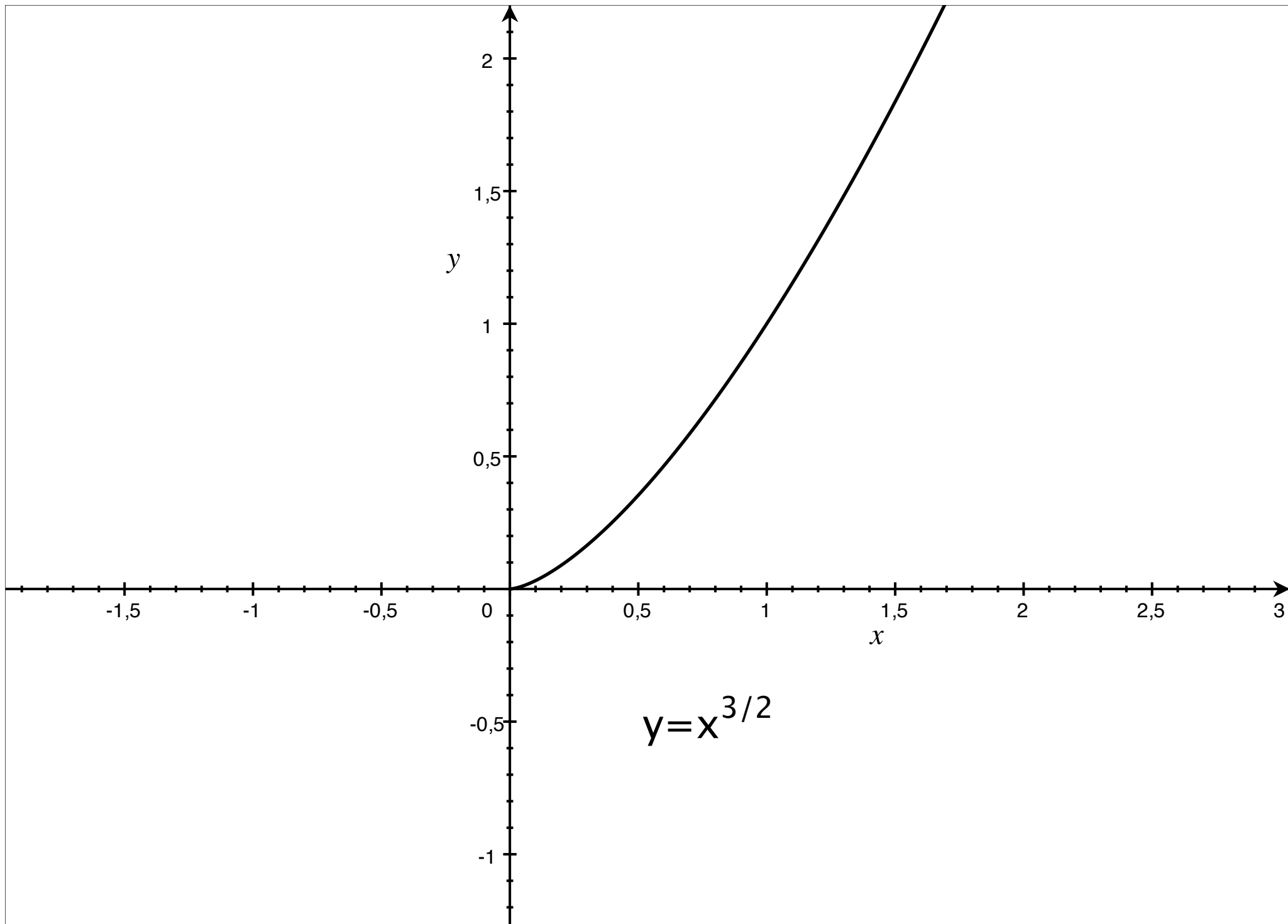
(la 9) è una facile conseguenza della 7)).

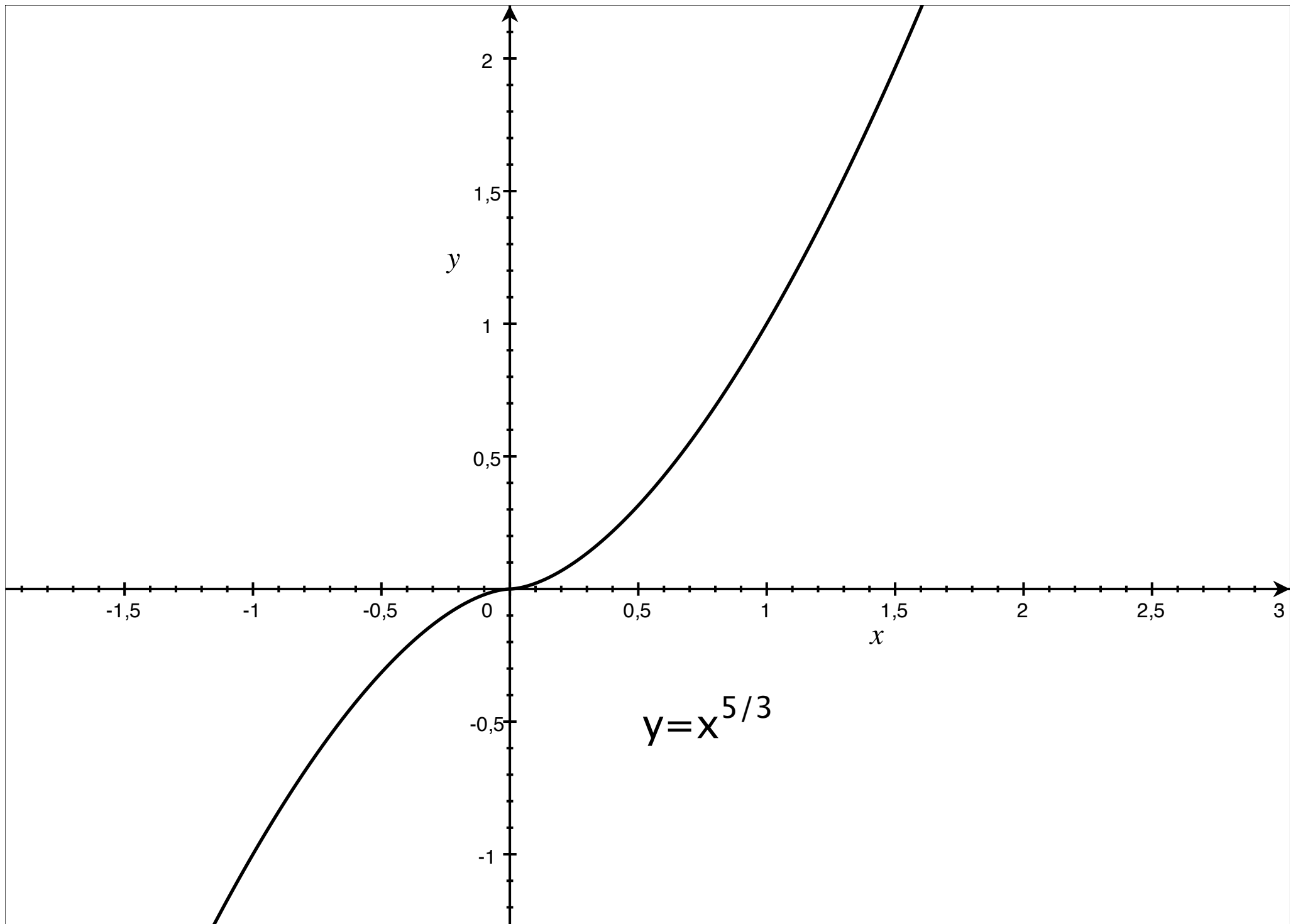


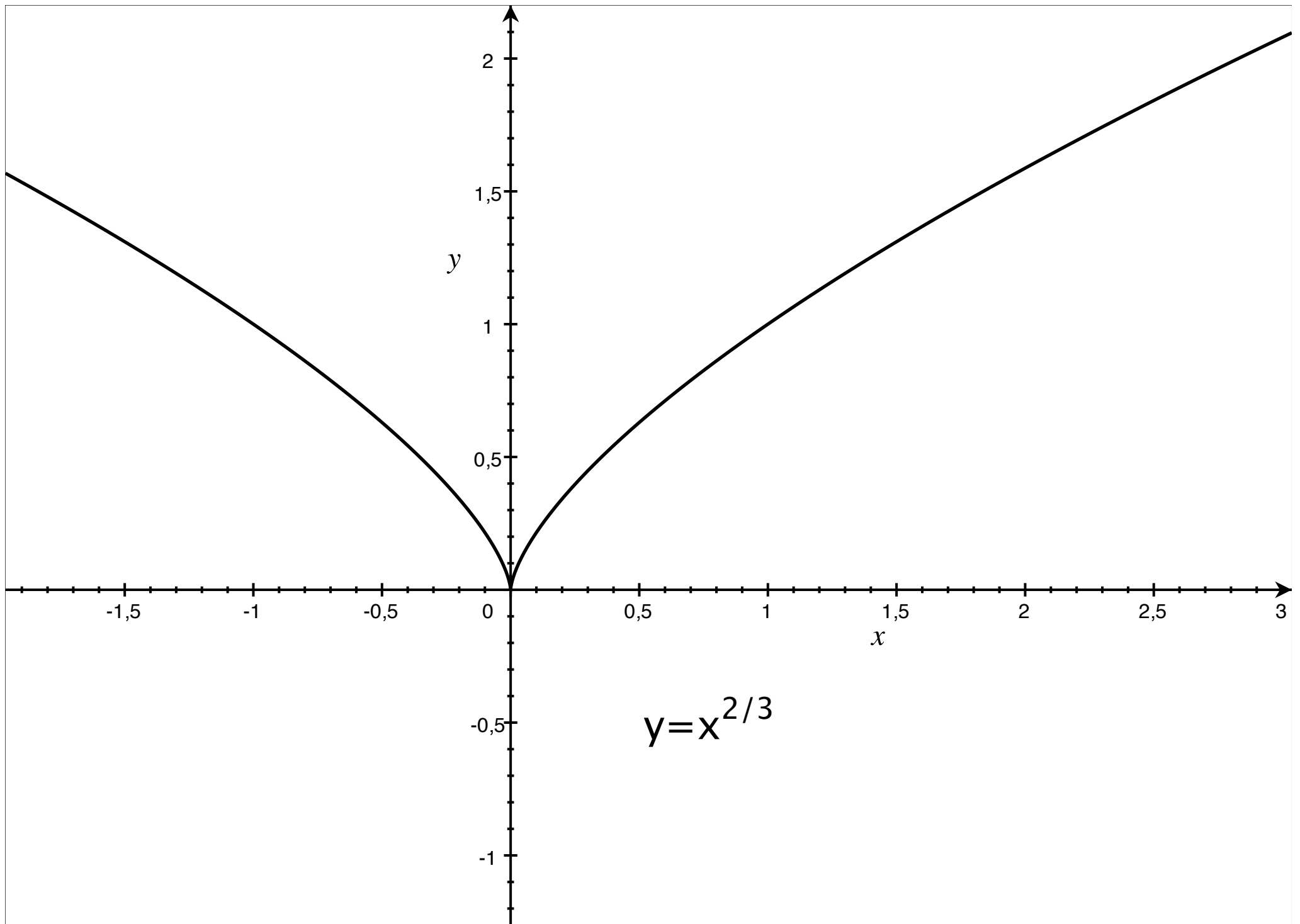


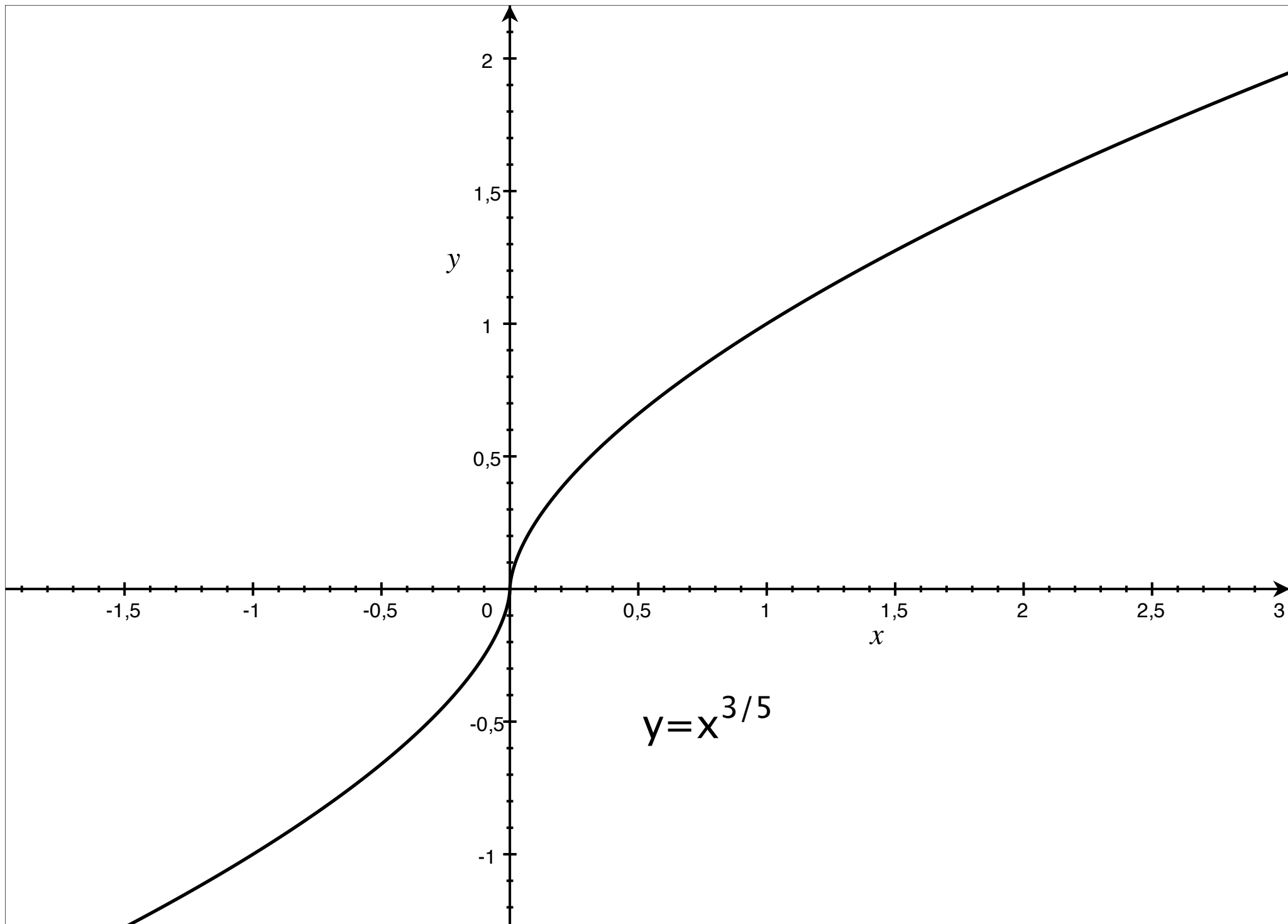


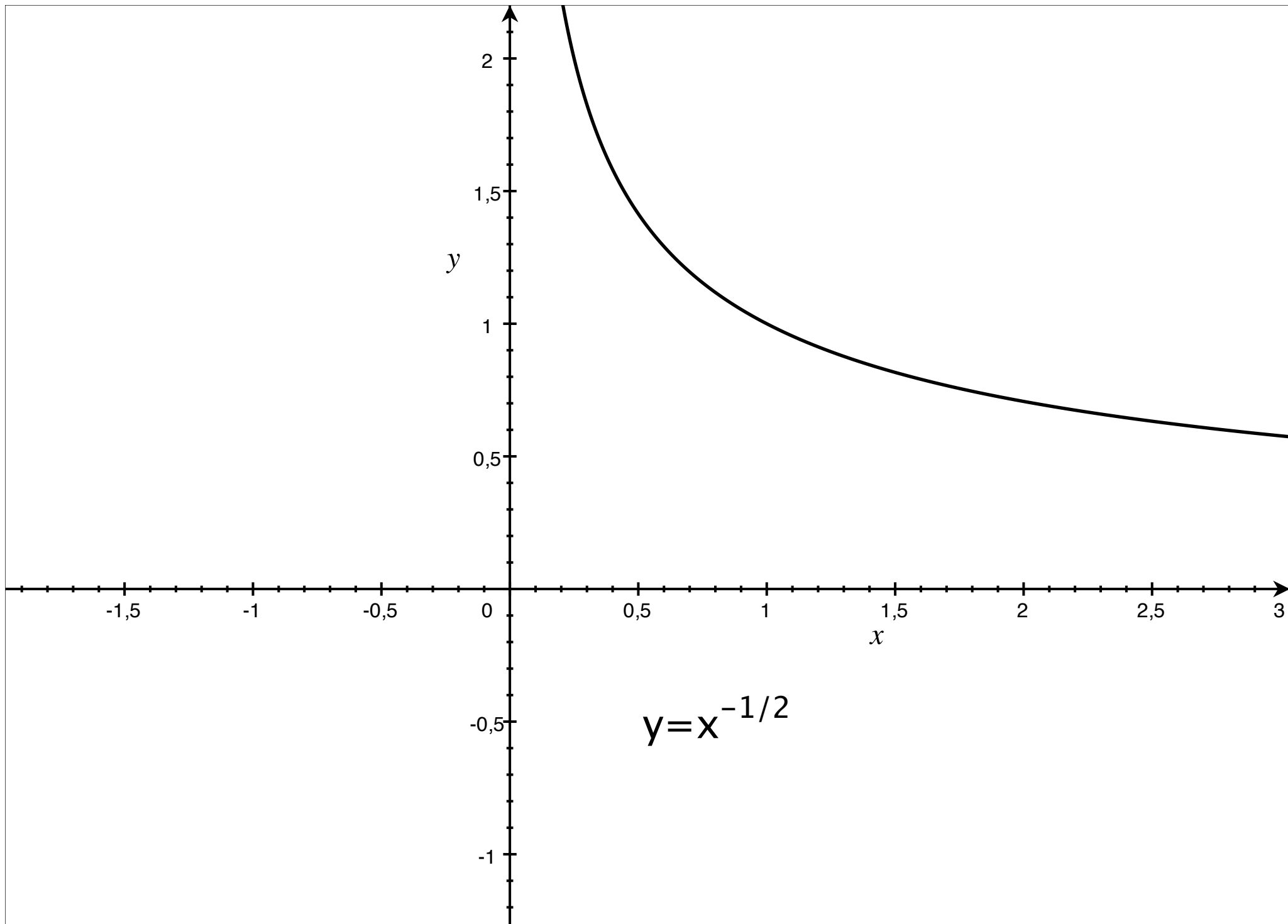


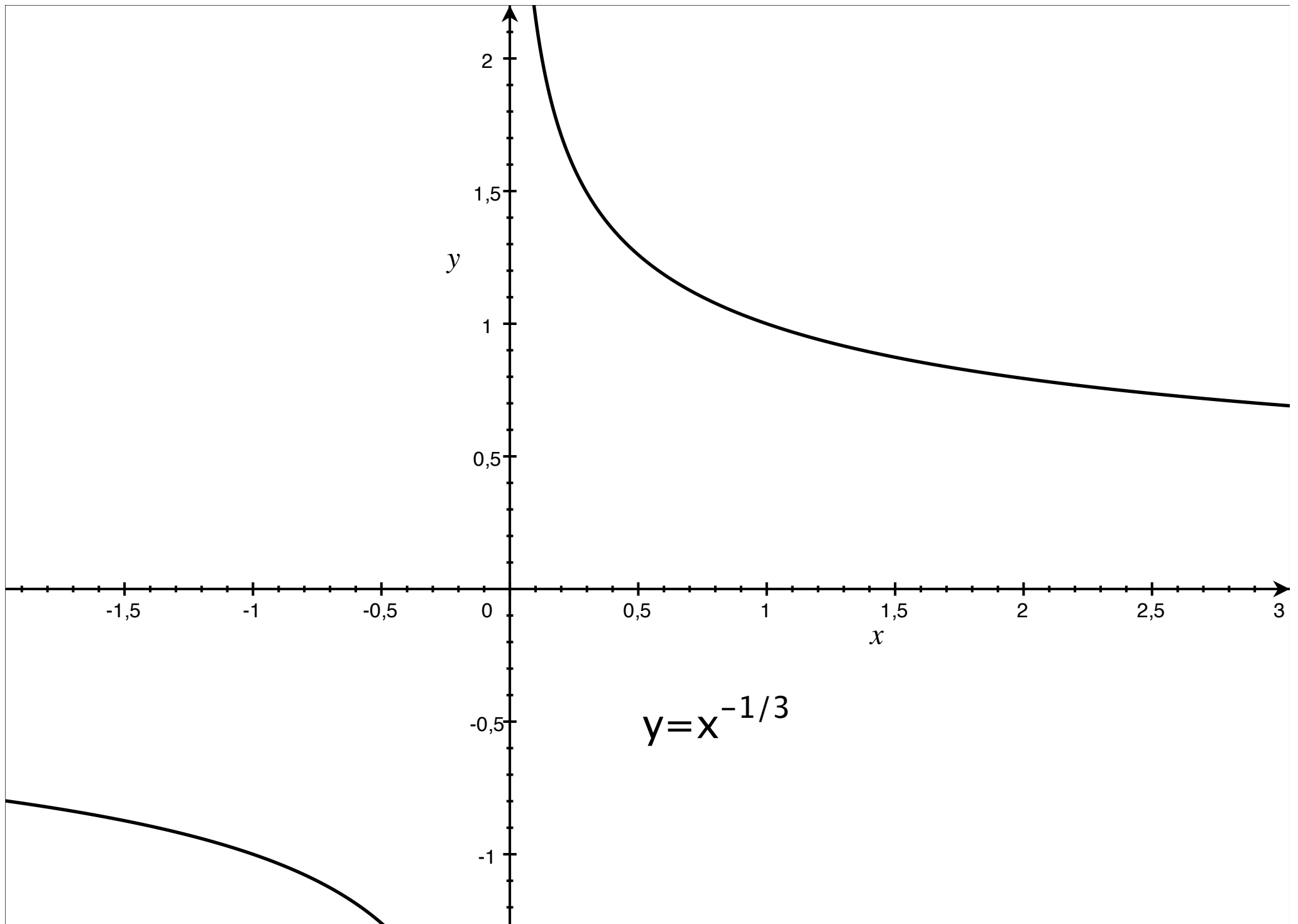


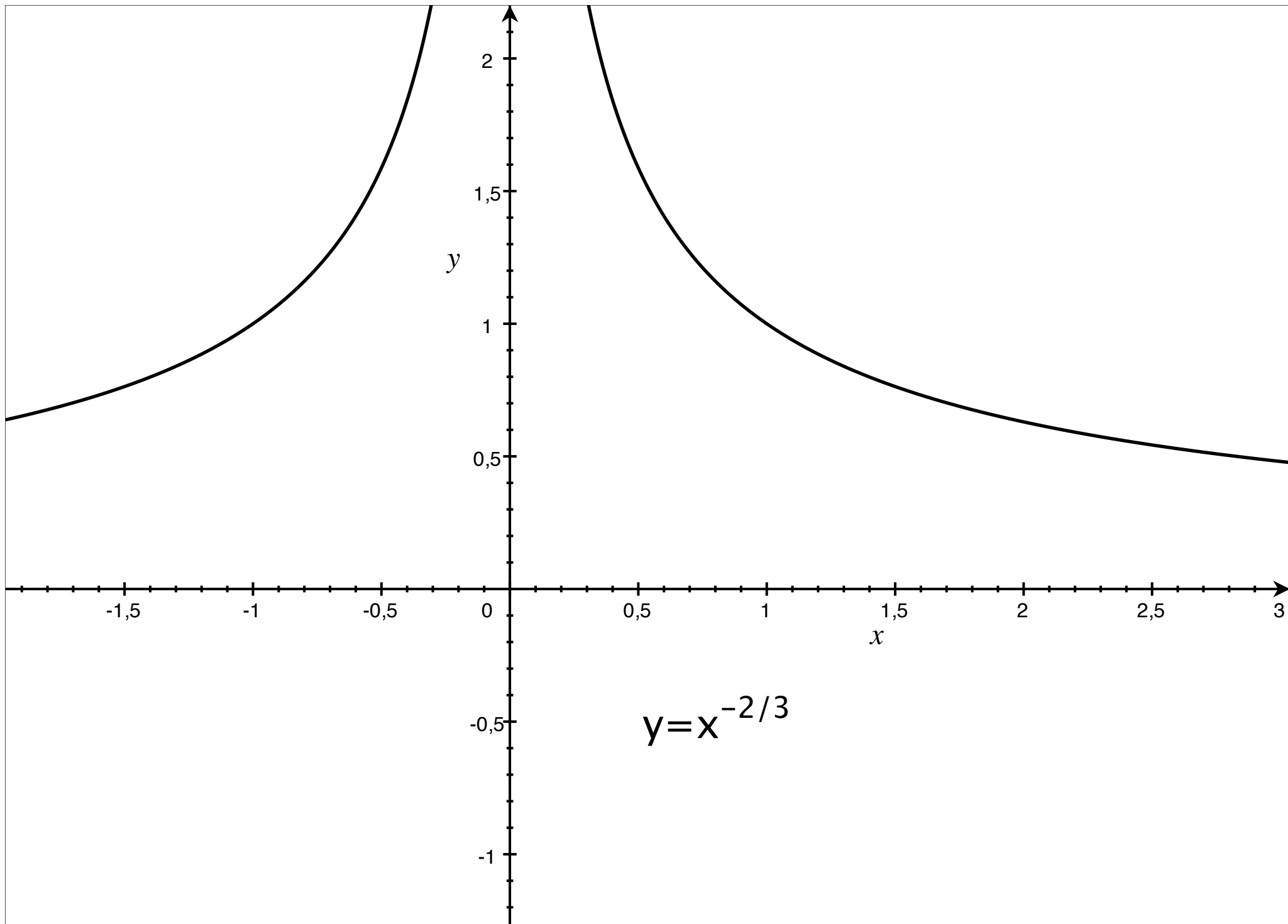


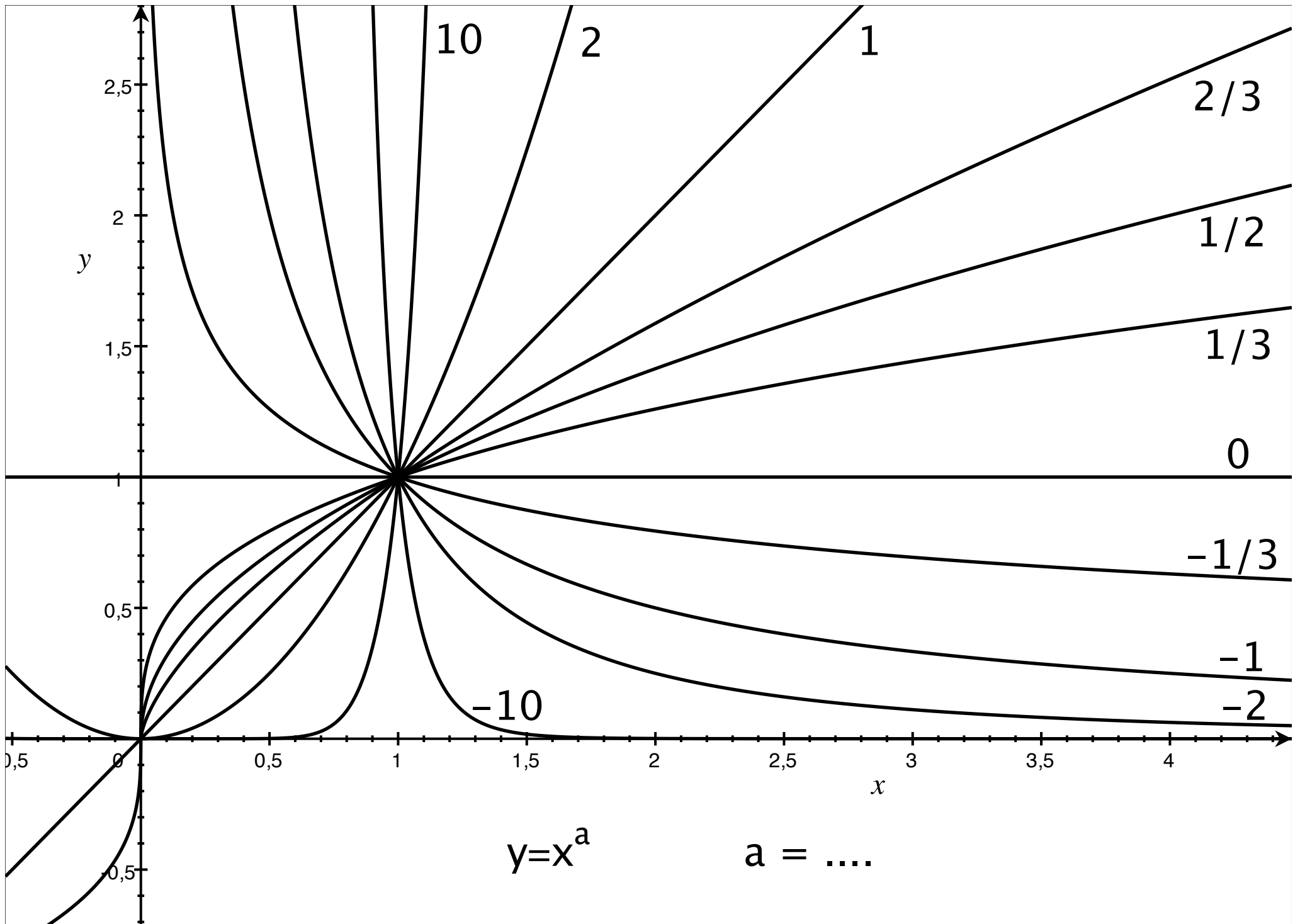


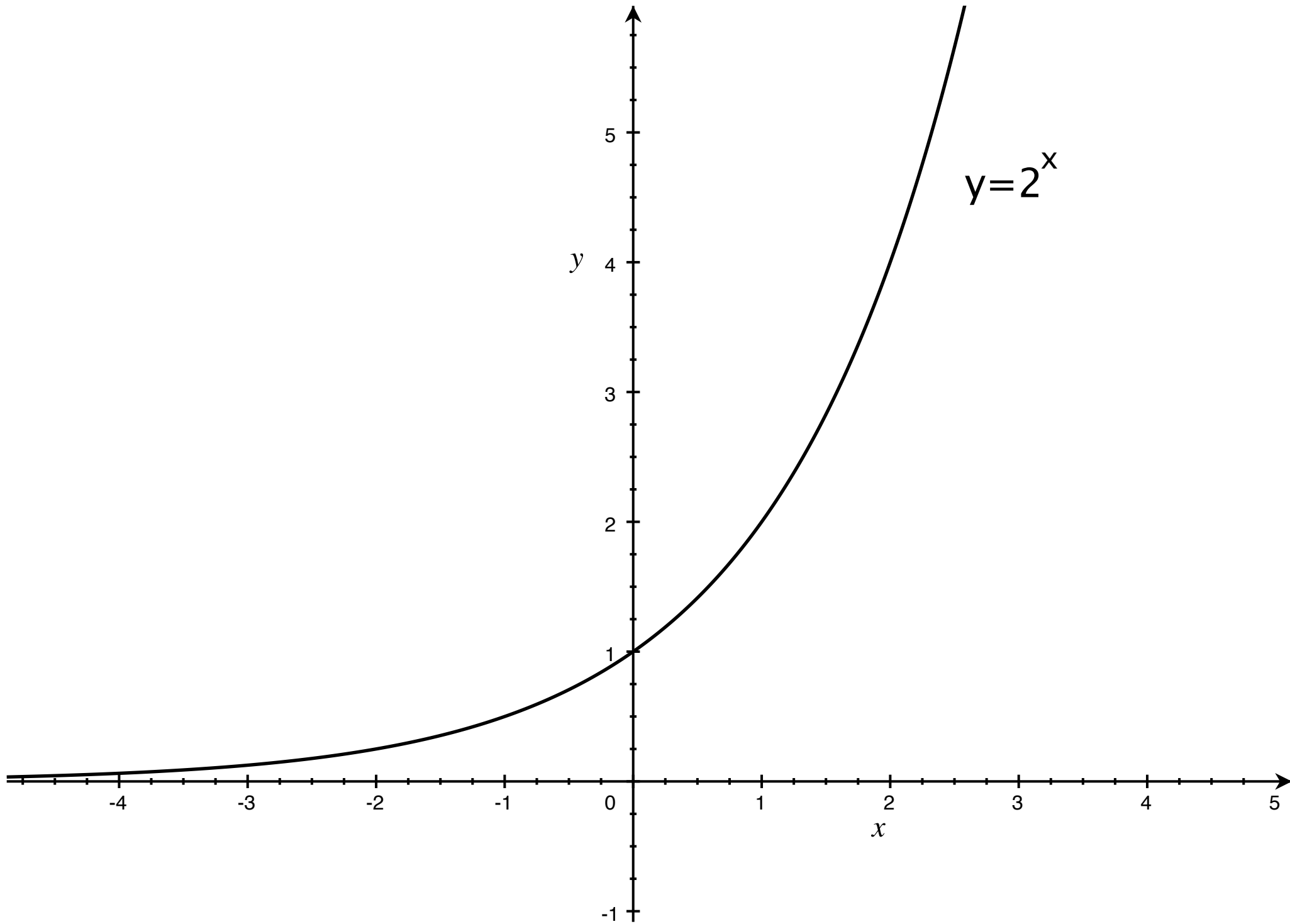


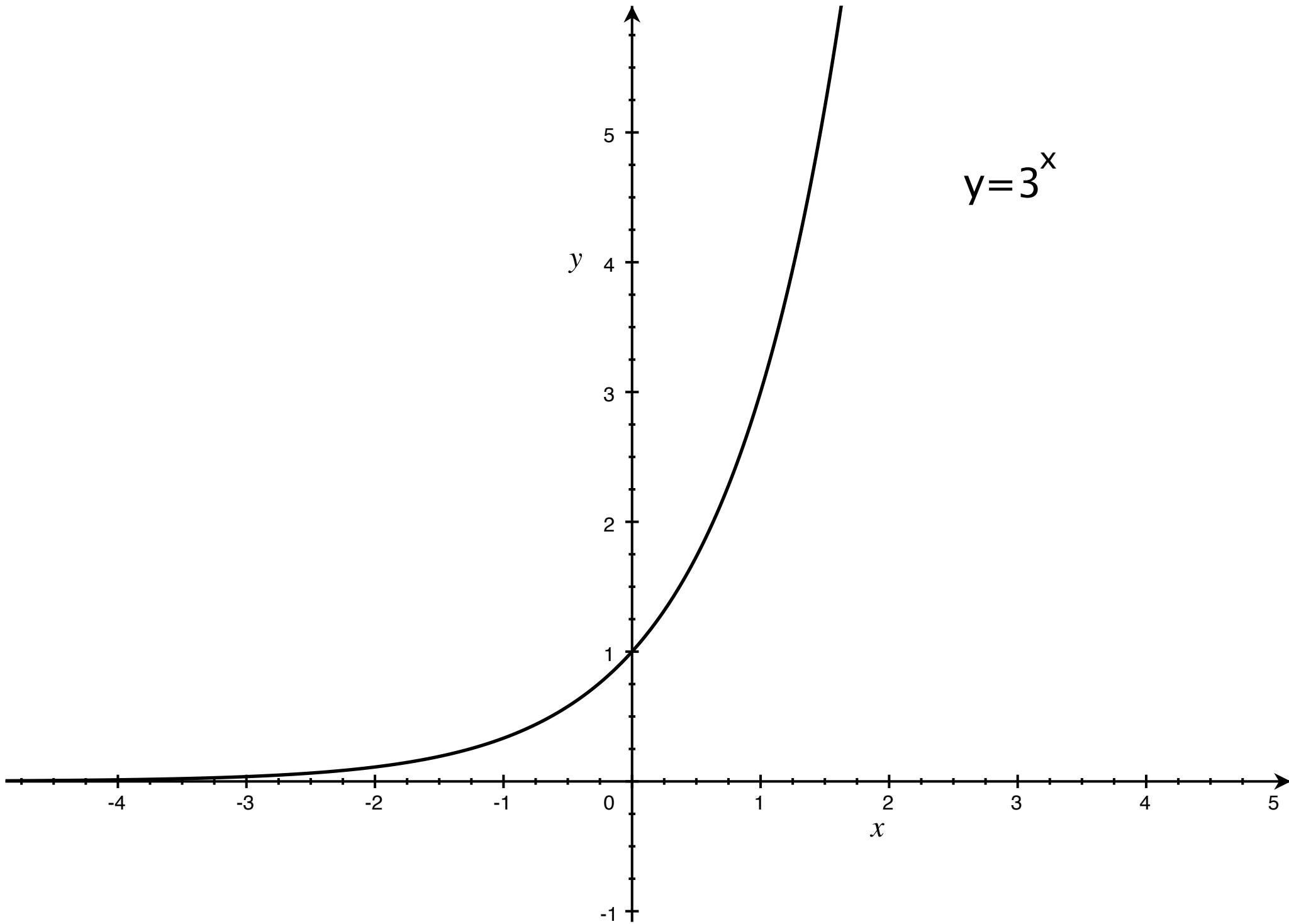


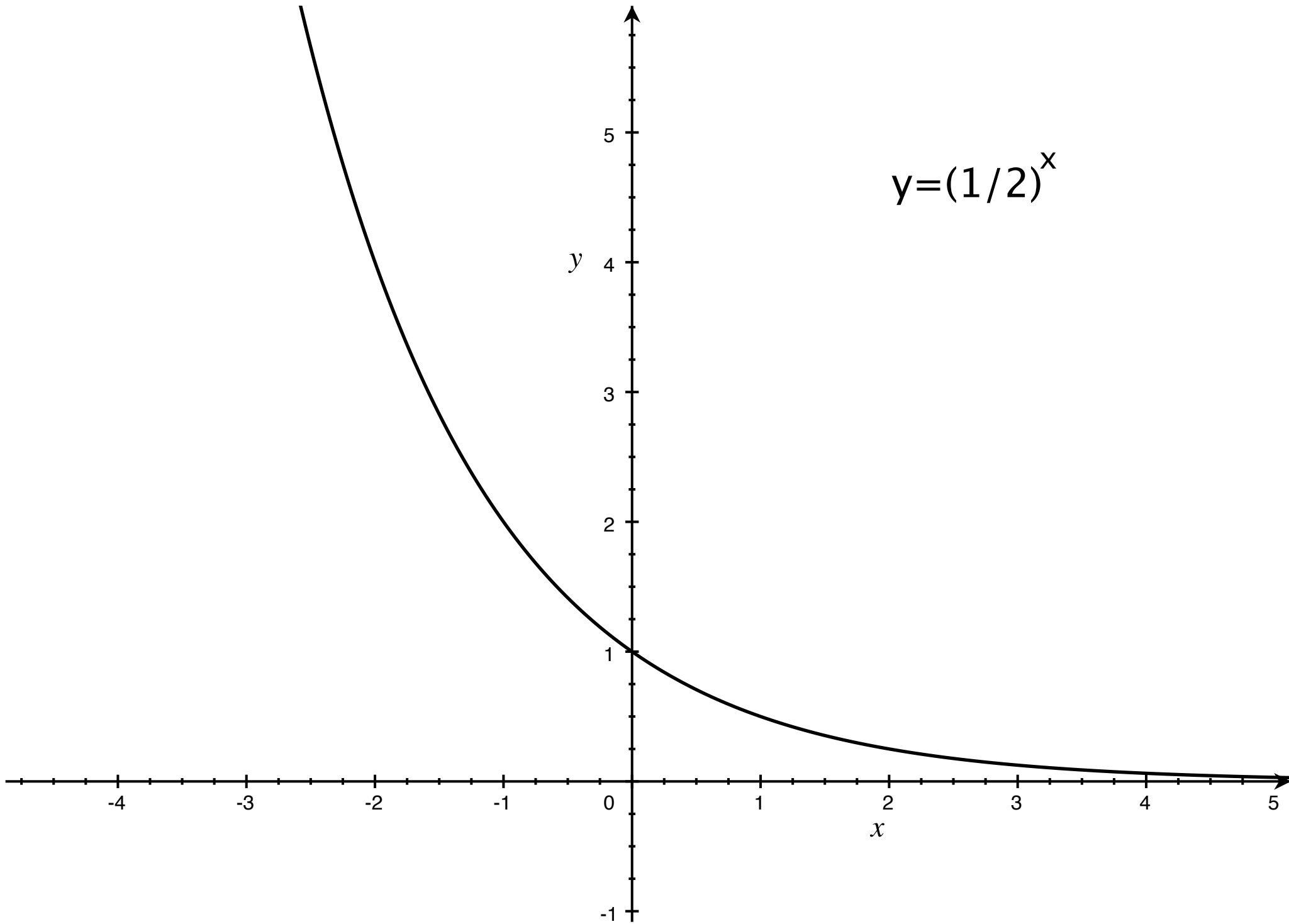


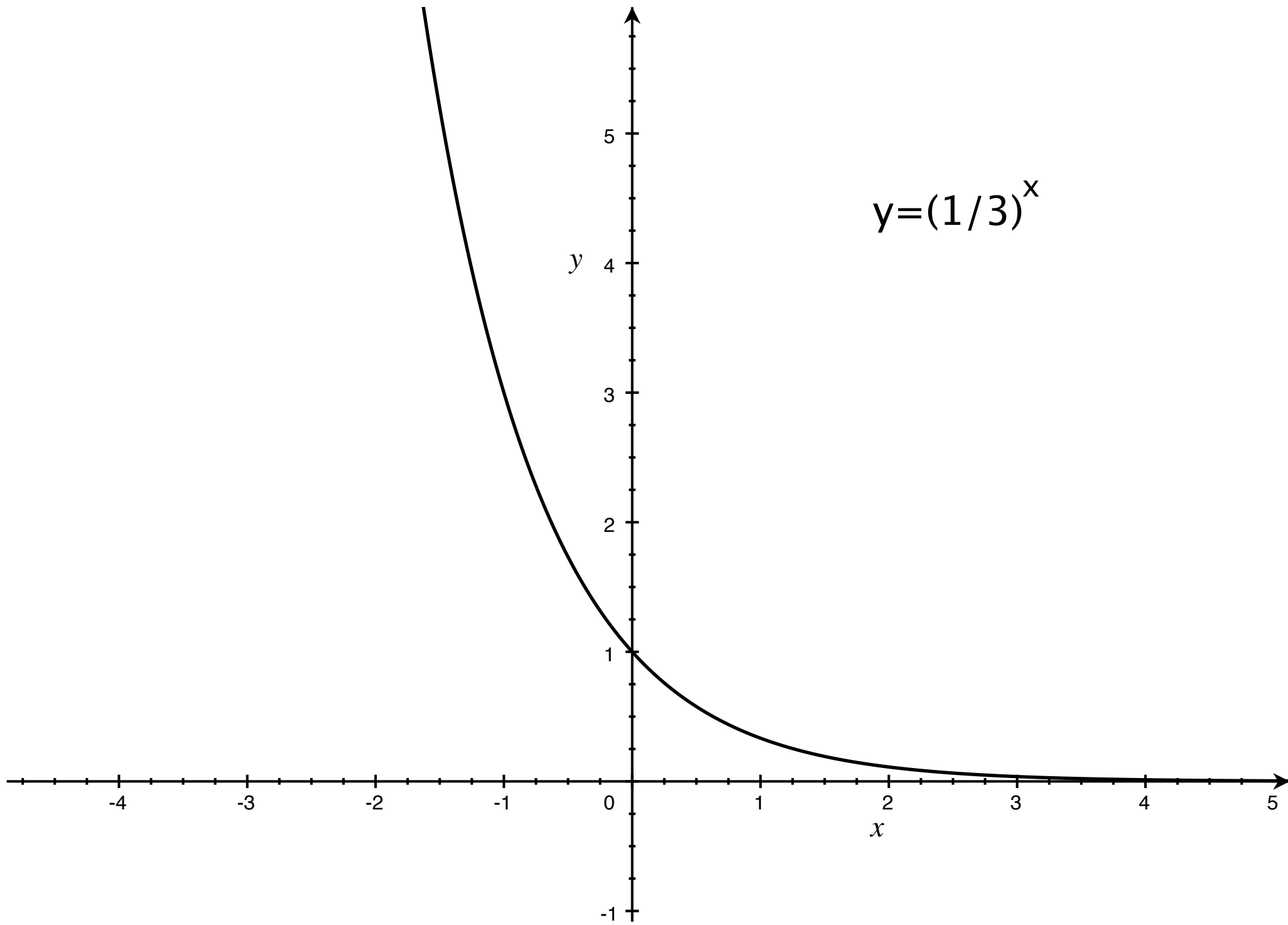


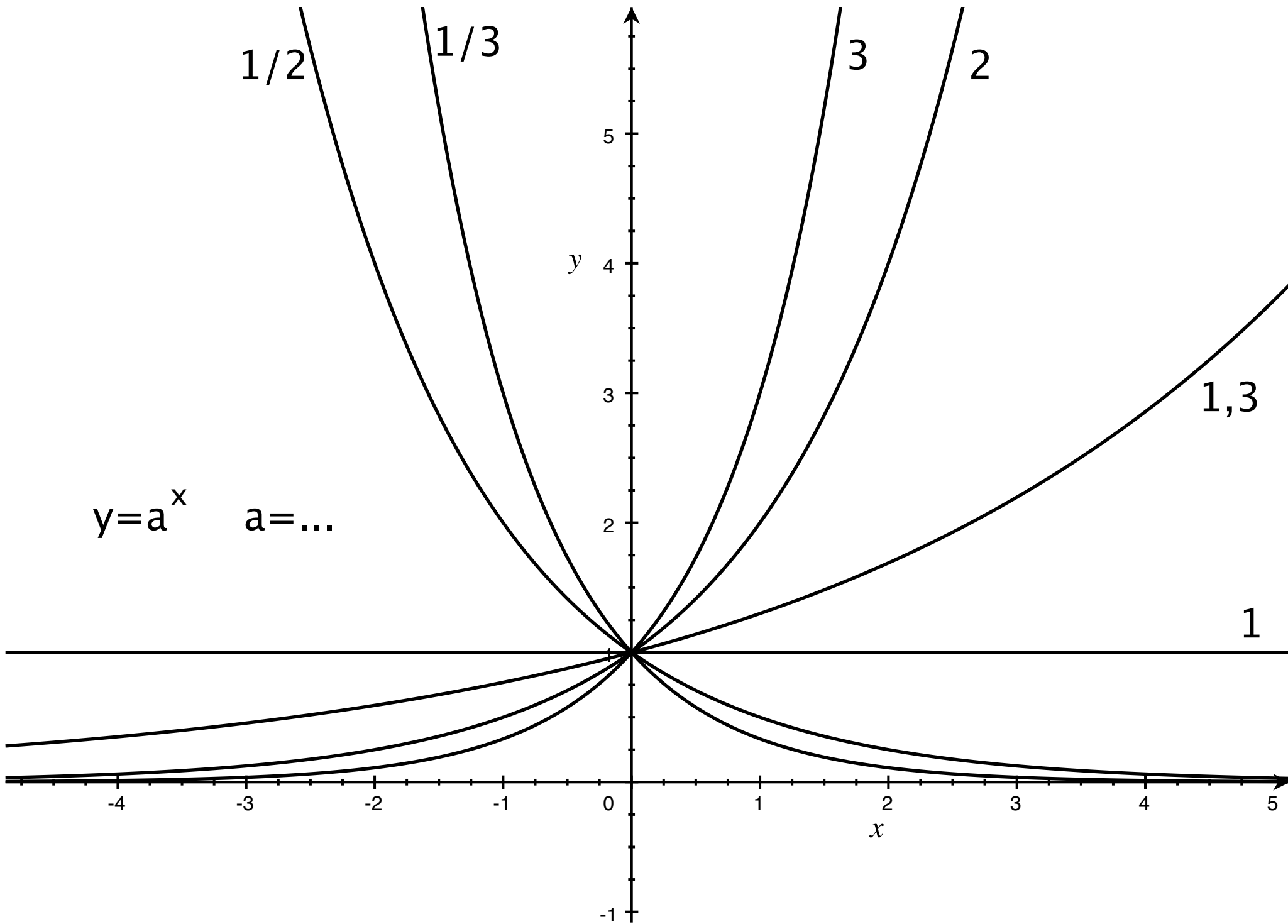


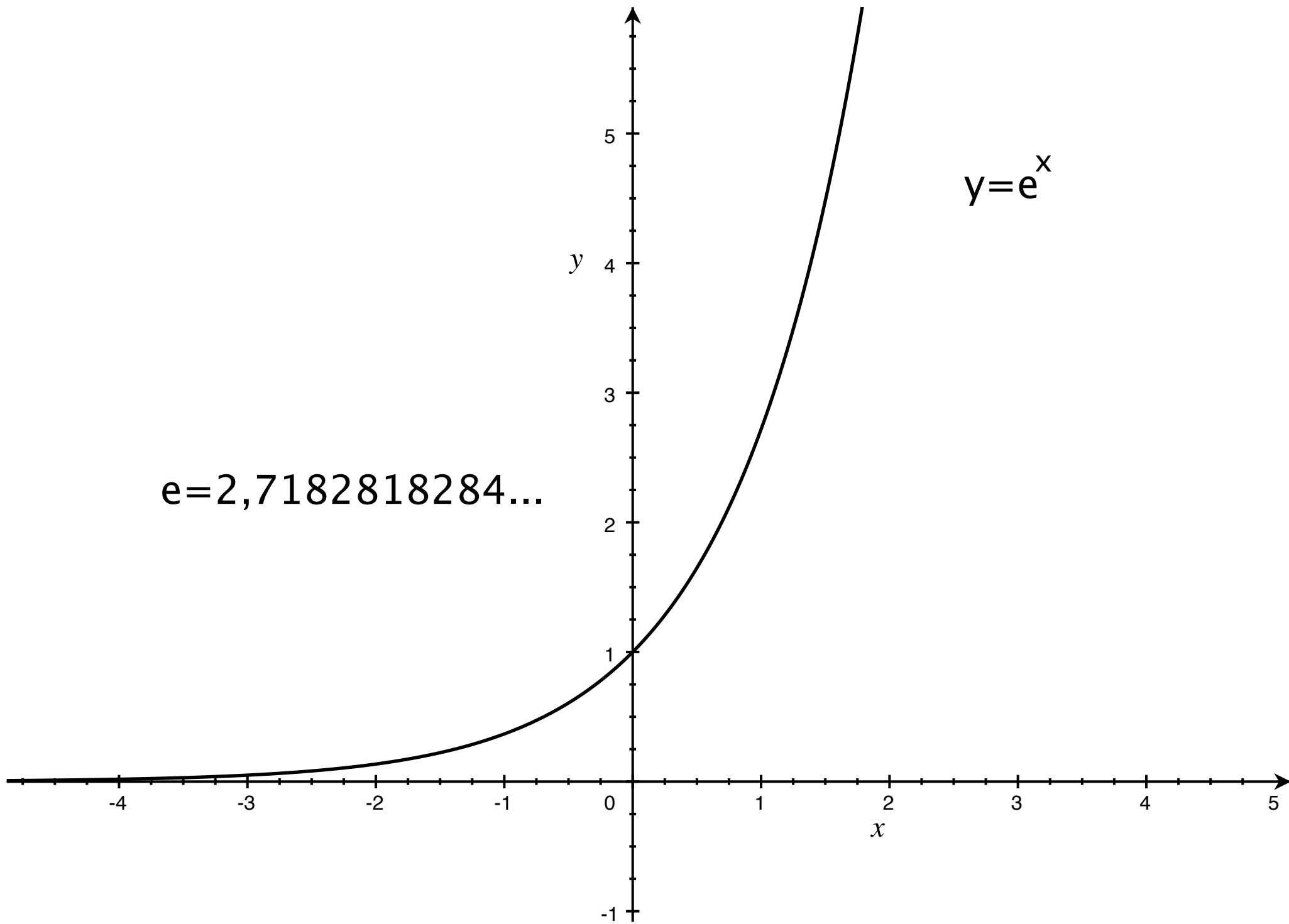












Proprietà Per ogni $a, b, x, y \in \mathbb{R}^+$, $a \neq 1$, $b \neq 1$, risulta:

1) $a^{\log_a x} = x$ (per definizione);

2) $\log_a xy = \log_a x + \log_a y$

3) $\log_a (1/x) = -\log_a x$

4) $\log_a (x/y) = \log_a x - \log_a y$

5) $\log_a x^\alpha = \alpha \log_a x \quad \forall \alpha \in \mathbb{R}$

$$6) \log_a x = 1/\log_x a = -\log_{\frac{1}{a}} x$$

$$7) \text{ (Cambiamento di base) } \log_a x = \log_b x / \log_b a$$

$$8) x > y > 0 \Rightarrow \begin{cases} \log_a x > \log_a y & \text{se } a > 1 \\ \log_a x < \log_a y & \text{se } 0 < a < 1 \end{cases}$$

$$9) \forall a \in \mathbb{R}^+, a \neq 1 \Rightarrow \begin{cases} \log_a 1 = 0 \\ \log_a a = 1 \\ \log_a (1/a) = -1; \end{cases}$$

$$10) \begin{cases} \forall x \neq 0 \Rightarrow \log_a x^2 = 2 \log_a |x| \\ \forall x, y : xy > 0 \Rightarrow \log_a (xy) = \log_a |x| + \log_a |y|. \end{cases}$$

